A Novel Equation for Calculating the Expansion Factor of Gas in Control Valve Sizing by considering the Z-factor

Nasser Faraz[1]*, Mohammad Reza Zohrabi[2], Maede Eslami[3]

*(ngh.res@gmail.com)

Abstract

In control valve sizing for compressible fluids, the gas flow rate versus the pressure drop ratio represents a straight line with constant slope at low pressure drop ratios, but as the pressure drop ratio increases, it deviates from linearity due to the variation of fluid density (expansion) as it passes through the valve and for the change in the area of vena contracta as the pressure drop varies. Due to this density reduction, the mass flow rate of a compressible fluid must therefore be lower than that of an incompressible fluid. Such an effect is taken into account by means of the expansion factor "Y" which is a key parameter in control valve sizing equations and makes it possible to use the incompressible sizing equations for compressible services. The current expansion factor equation that has been widely accepted in gas industry has been defined as a function of pressure drop ratio, fluid type and valve geometry. In this definition the effect of gas composition (especially for natural gas) and molecules interactions on the expansion factor has not been considered whereas for a real gas the expansibility and compressibility is also depend on the gas compressibility factor "Z" and molecules interactions. So the main objective of this paper is to derive a more general equation that considers the effect of attractive and repulsive forces of gas molecules on the expansion factor by using Z-factor. This equation is the solution of a differential equation which was solved with several substitutions. The comparison of these equations for the same value of pressure drop ratio shows that the new equation represents lower values of expansion factor compared to the old equation if attractive forces dominate repulsive forces. Also if repulsive forces dominate attractive forces, the new one shows higher values of expansion factor. Another important result of this new equation is that the expansion factor can be less than 0.667 in contrast to the old one.

Key words: Control valve Sizing, Expansion factor, Gas compressibility factor

[1] National Iranian South Oil Company
[2] National Iranian South Oil Company
[3] National Iranian South Oil Company

1. Introduction

Improper valve sizing can be both expensive and inconvenient. A valve that is too small will not pass the required flow, and the process will be starved. An oversized valve will be more expensive, and it may lead to instability and other problems. So it is very important to size the valve precisely [1].

Using the principle of conservation of energy, Daniel Bernoulli found that as a liquid flows through an orifice, the square of the fluid velocity is directly proportional to the pressure differential across the orifice and inversely proportional to the specific gravity of the fluid. By taking into account units of measurement, the proportionality relationship previously mentioned, energy losses due to friction and turbulence, and varying discharge coefficients for various types of orifices (or valve bodies), a basic liquid sizing equation can be written as follows [1, 2]:

$$Q = C_V \sqrt{\Delta P / SG} \qquad \text{1-1}$$

Where, "Q" is capacity in *gallons per minute*, "Cv" is valve sizing coefficient determined experimentally for each style and size of valve, using water at standard conditions as the test fluid, "ΔP" is pressure differential in *psi* and "SG" is specific gravity of fluid ($SG_{water} = 1$). Thus, Cv is numerically equal to the number of U.S. gallons of water that will flow through the valve in one minute when the pressure differential across the valve is one pound per square inch. Cv varies with both size and style of valve, but provides an index for comparing liquid capacities of different valves under a standard set of conditions [1].

The basic liquid sizing equation tells us that the liquid flow rate through a control valve has linear relationship with the square root of pressure drop. This linear relationship does not always hold true. As the pressure drop is increased, the flow reaches a point where it no longer increases. Once this happens, additional increases in pressure drop across the valve do not result in additional flow, and flow is said to be choked [1].

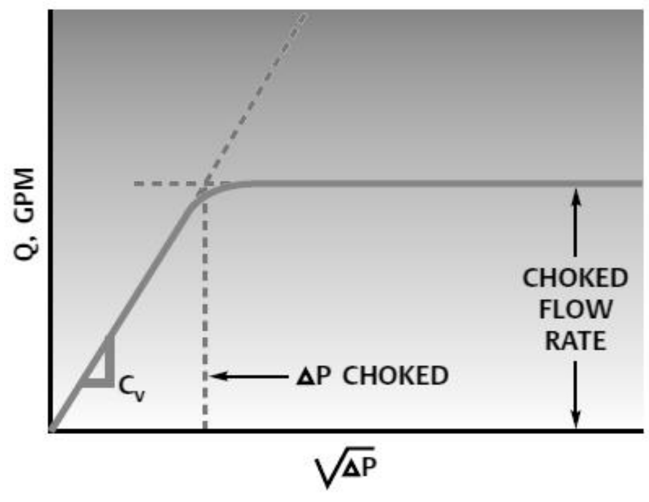

Figure 1: Liquid flow rate vs square root of pressure drop [1].

The equation for gas flow is almost identical to the equation we would use for liquid in cases where flow, "Q_m", was given in *pounds per hour*. Note that the subscript 1, for pressure and density, indicates that they are the upstream conditions of the valve. The only difference is that instead of using the square root of pressure drop "ΔP" in the equation, it is used the square root of the Pressure drop ratio, "$x = \Delta P/P_1$" [2, 3].

$$Q_m = 63.3C_V\sqrt{xP_1\rho_1} \quad 1\text{-}2$$

At low pressure drop ratios the flow follows the straight line, but then it deviates more and more (as shown in figure 2, the flow rate is no longer proportional to the square root of the pressure differential as in the case of incompressible fluids) until at last, further increases in pressure drop ratio do not yield any additional flow. At this point we say that flow has become choked [2].

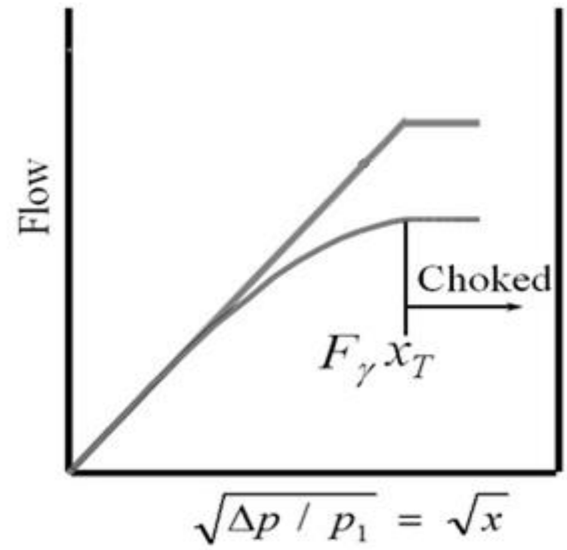

Figure 2: Gas mass flow rate vs square root of pressure drop ratio [2].

The principle difference between the nature of the flow of gas and the flow of liquid through control valves is that liquids are incompressible and gasses are compressible. When the pressure of a liquid changes, the volume and density, ρ, remain unchanged, while on the other hand, pressure change in a gas result in density change. Due to this density reduction the gas is accelerated up to a higher velocity than the one reached by an equivalent liquid mass flow. Under the same ΔP the mass flow rate of a compressible fluid must therefore be lower than the one of an incompressible fluid. So to accurately size control valves for gas service, an *expansion factor*, "*Y*", is added to the equation 1-2, to correct the calculated flow (and the graph) for both changes in density at the vena contracta and for vena contracta enlargement [2, 3].

$$Q_m = 63.3YC_V\sqrt{xP_1\rho_1} \quad 1\text{-}3$$

2. Expansion Factor

1.1 Conventional method [4]

The expansion factor Y allows using for compressible fluids the same equation structure valid for incompressible fluids. It has the same nature of the expansion factor utilized in the equations of the throttling type devices (orifices, nozzles or venturi) for the measure of the flow rate.

The Y's equation is obtained from the theory on the basis of the following hypothesis that is experimentally confirmed:

- Y is a linear function of $x = \Delta P/P_1$;
- Y is function of the geometry (i.e. type) of the valve;
- Y is a function of the fluid type, namely the exponent of the adiabatic transformation $\gamma = Cp/Cv$.

From the first hypothesis:

$$Y = 1 - ax \qquad \text{2-1}$$

Therefore:

$$Q_m \propto Y\sqrt{x} = (1 - ax)\sqrt{x} = \sqrt{x} - a\sqrt{x^3} \qquad \text{2-2}$$

To calculate the value of Y which makes Q_m maximum, the derivative of above function must be calculated, (this means finding the point where the rate "dQ_m / dx" becomes zero):

$$\frac{dQ_m}{dx} = \frac{1}{2\sqrt{x}} - \frac{3a\sqrt{x}}{2} = 0$$
$$\frac{1}{\sqrt{x}} = 3a\sqrt{x} \Rightarrow x = \frac{1}{3a} \qquad \text{2-3}$$

i.e.: $Y = 1 - ax = 1 - \left(\frac{1}{3a}\right)a = \frac{2}{3}$ 2-4

Therefore when $x = 0$, $Y = 1$ and when the flow rate is maximum $Y = 2/3$, (i.e. $x = x_T$), so the equation of Y becomes the following:

$$Y = 1 - \frac{x}{3x_T} \qquad \text{2-5}$$

Thus the second hypothesis is taken into account. In fact, x_T is an experimental value to be determined for each valve type.

Finally the third hypothesis will be taken into account with an appropriate correction factor, the specific heat ratio factor Fγ, which is the ratio between the exponent of the adiabatic transformation for the actual gas and the one for air:

$$F_\gamma = \frac{\gamma}{1.4} \quad 2\text{-}6$$

The final equation becomes:

$$Y = 1 - \frac{x}{3F_\gamma x_T} \quad 2\text{-}7$$

Therefore the maximum flow rate is reached when $x = F_\gamma x_T$, that means the expansion factor reaches the minimum value of 0.667.

1.2 New approach

As mentioned earlier the expansion factor is a function of pressure drop ratio, valve geometry and fluid type. But an important parameter that was not considered in the conventional method is molecular interaction. The expansion of gas is influenced by attractive and repulsive forces of molecules that can be vary in different pressure drop ratios, and a parameter that can determine the domination of these forces is gas compressibility factor, "Z".

From the fluid properties, the equation 2-8 shows the relationship between density and isothermal compressibility:

$$C_p = \frac{1}{\rho}\frac{\Delta\rho}{\Delta P} \Rightarrow \frac{\rho_2}{\rho_1} = 1 - C_p \Delta P \quad 2\text{-}8$$

Also for a constant volume, (V = cte):

$$\frac{\rho_2}{\rho_1} = \frac{m_2}{m_1} = 1 - C_p \Delta P \quad 2\text{-}9$$

The above equation is another form of expansion factor, Y, and shows that when the downstream pressure is lower than upstream ($\Delta P>0$), downstream mass and density decrease. If the "ΔP" be substituted with "xP_1", then:

$$Y = 1 - (C_p P_1)x \quad 2\text{-}10$$

The above equation is expansion factor without valve type effect. By considering the equation 2-1, we have the following equation with valve type effect:

$$Y = 1 - a(C_p P_1)x \quad 2\text{-}11$$

As it is seen, If the gas assumed to be ideal, "$C_p = 1/P$", the term "C_pP_1" will drop from the equation 2-11 and give the equation 2-1. So it is concluded that the equation 2-1 and 2-11 is for ideal and real gas respectively. Also for "C_p" of a real gas we have:

$$C_p = \frac{1}{P} - \frac{1}{Z}\left(\frac{\Delta Z}{\Delta P}\right) \qquad 2\text{-}12$$

If "$\zeta = \Delta Z/Z_1$" and "$xP_1 = \Delta P$", therefore:

$$C_p = \frac{1}{P_1} - \frac{1}{P_1}\left(\frac{\zeta}{x}\right) = \frac{1}{P_1}\left(1 - \frac{\zeta}{x}\right) \qquad 2\text{-}13$$

Then:

$$Q_m \propto Y\sqrt{x} = \{1 - a(C_pP_1)x\}\sqrt{x} = (1 - ax + a\zeta)\sqrt{x} \qquad 2\text{-}14$$

To calculate the value of Y which makes Q_m maximum, the derivative of above function must be calculated, (this means finding the point where the rate "dQ_m / dx" becomes zero):

$$\frac{dQ_m}{dx} = \frac{1}{2\sqrt{x}} - \frac{3a\sqrt{x}}{2} + \left(\frac{a\zeta}{2\sqrt{x}} + a\sqrt{x}\,\frac{d\zeta}{dx}\right) = 0 \qquad 2\text{-}15$$

Therefore we have:

$$a\frac{d\zeta}{dx} - \frac{3a}{2} + \frac{a\zeta + 1}{2x} = 0 \qquad 2\text{-}16$$

So we have to solve the above differential equation to obtain the value of Y which makes Q_m maximum, ($x = x_T$, $\zeta = \zeta_T$):

$$a = \frac{1}{c/\sqrt{x_T} + x_T - \zeta_T} \qquad 2\text{-}17$$

i.e.: $Y = 1 - ax_T + a\zeta_T = \dfrac{ac}{\sqrt{x_T}}$ 2-18

Therefore when $x = 0$, $Y = 1$ and when the flow rate is maximum Y, (i.e. $x = x_T$), will be:

$$Y = \frac{c}{c + x_T\sqrt{x_T} - \zeta_T\sqrt{x_T}} \qquad 2\text{-}19$$

So the general form of Y becomes the following:

$$Y = 1 - \frac{x - \zeta}{c/\sqrt{x_T} + x_T - \zeta_T} \qquad 2\text{-}20$$

Finally by considering the specific heat ratio factor Fγ, we have:

$$Y = 1 - \frac{x - \zeta}{c/\sqrt{x_T F_\gamma} + x_T F_\gamma - \zeta_T} \qquad 2\text{-}20$$

In the above equations, "*c*" is a constant that is independent of pressure drop ratio "*x*" and must be determined by experiment.

3. Case Study

In this section, the above approach is used for one of Iranian gas plants and the results are presented. The properties of control valve and flowing fluid are given in the following tables:

Table 1: Fluid and Valve properties in case study.

P_1 (KPa)	P_2 (KPa)	T (K)	x_T	SG	T_c (K)	P_c (KPa)	γ
680	310	433	0.6	1.5	304	7377	1.3

The value of expansion factor was calculated from both methods and the results are shown in table 2:

Table 2: The results of old and new methods for expansion factor.

Variables	Eq 2-20 (New method)	Eq 2-7 (Old method)
x	0.544	0.544
F_γ	0.928	0.928
ζ	-0.00616	–
ζ_T	0.9955	–
c *	$2(X_T)^{1.5}$	–
Y	0.316	0.697

The value of "*c*" in new method should be determined by experiment, but in the absence of experimental data it can be consider $2(X_T)^{1.5}$.

4. Conclusion

The comparison of conventional method and new approach for the same value of pressure drop ratio shows that the new approach represents lower values of expansion factor compared to the conventional method if attractive forces dominate repulsive forces. Also if repulsive forces dominate attractive forces, the new approach shows higher values of expansion factor. Another important result of the new approach is that the expansion factor can be less than 0.667 (depend on the value of "*c*"), as it was shown in case study, in contrast to the conventional method.

Acknowledgment

Authors thank *National Iranian Gas Company (NIGC)*, *National Iranian Oil Company (NIOC)* and *National Iranian South Oil Company* for their help and financial support.

Nomenclature

Symbol	***Description***	***Units***
C_v	*Valve coefficient*	*U.S. gallons/min*
x	*Pressure drop ratio*	*dimensionless*
xT	*Pressure drop ratio factor in choked flow condition*	*dimensionless*
Y	*Expansion factor*	*dimensionless*
γ	*Specific heat ratio*	*dimensionless*
F_γ	*Specific heat ratio factor,* $= \gamma/1.4$	*dimensionless*
ζ	*Gas compressibility ratio*	*dimensionless*
ζ_T	*Gas compressibility ratio factor in choked condition*	*dimensionless*
c^*	*Constant coefficient, valve type dependent*	*dimensionless*

References

[1] Fisher Controls International, (2005), "*Control Valve Hand Book*", 4th Edition

[2] F. M. Cain and B. G. Lipták, (1933), "*Control Valve Selection and Sizing*"

[3] ISA-75 (60534-2-1 Mod), (2007), "*Flow Equations for Sizing Control Valves*"

[4] Parcol S.P.A (2013), "*Hand Book for Control Valve Sizingr*"-Technical Bulletin