MAINTENANCE OPTIMIZATION FOR GAS TURBINES BASED ON SUPPORTABILITY ANALYSIS

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Abstract

Integrated logistic support ILS is a methodology to identifying and optimizing maintenance resources in order to preserve a desired level of system performances. Its successful implementation relies on the development of structured support methods for complex systems by which different maintenance activities are predefined and optimized. Examples of these methods are spare part management, repair facilities management, reliability centred maintenance, among others. This paper presents a supportability decision model, as application of ILS, based on queuing theory for managing spares inventory in a limited repair capacity. Validation was performed by real world testing of the model within the Algerian petroleum industry. Such a maintenance support model potentially reduces system downtimes and leads to higher system output. The results show that gas turbines, major equipment in gas boosting stations, could be effectively managed by minimizing their whole life costs and maximizing their outputs. The proposed model is adequately generic to be extended to other complex systems in the Petroleum Industry.

Keywords: spare part, repair capacity, queuing, availability, logistics support & maintenance.

INTRODUCTION

Today's asset management developments reveal a recent intensive partnership between the different business actors. Corporation relationships between manufacturers and clients have become more common; there are industry guidelines for more integrated management approaches such as: integrated logistics support for the military industry (United States Department of Defense, 1983), Private Finance Initiative PFI for construction industry (Kishk et al., 2003), among others. Accordingly, more organisations are adopting a holistic based decision-making that relates design, manufacturing and operation phases. Within this new environment, clients are requiring more reliable products along with an efficient maintenance support. (Blanchard, et al., 1998 and Rappold, et al., B. D, 2009) asserted that maintenance and its support represent the major contributor to whole life cost for many types of systems. To this end the integrated logistic support ILS, which is a methodology to identifying and optimizing maintenance resources in order to preserve a desired level of system performances, plays important role in achieving these requirements.

Another actual tendency is characterised by the fact that current technological equipment such aircraft, HVAC, petroleum, medical and military equipment are becoming more complex and scattered over a huge geographical area (Sleptchenko, et al., 2005; and Rustenburg, et al., 2001). Besides, they have complex structures that malfunction because the enclosing items are either failed or worn out during operation. One way to ensure a high level of system availability is to hold enough spare parts to provide an immediate replacement of the failed items. Nevertheless, holding enough spares may be very costly and risks being obsolete over time; thus a balance between cost of spares and system availability is necessary. These issues are already challenging for systems consisting of thousands of items structured in several levels called the multi-indenture systems. In addition, these systems may be installed at different locations, in which case maintenance facilities should be needed at the local levels, intermediate levels and the central level: this is called the multi-echelon repair network (Kim, et al., 2007). The spare part allocation is, therefore, an optimal supply throughout all pyramidal subordination of maintenance levels. This optimisation has been regarded as an important area for maintenance cost reduction and has been considered in the last decades by many researchers (Kim, et al., 2000).

For literature on spare part optimisation, the evolution of the related models can be found in Sherbrooke, (1668), Kennedey, et al., (2002), Avsar, et al. (2000) and Sleptchenko, et al., (2002). In these papers the area of study is devoted to multi-echelon inventory systems in which spare part is stored at different levels. In addition, this bulk of research in multi-echelon spare part inventory management can be categorised into two main classes: spare part optimisation under infinite repair capacity and under limited repair capacity. However, these two classes are based on METRIC model developed by Sleptchenko, et al., (2002). In this model, also called Multi Echelon Technique Repairable Item Control, all repair levels are supplied by intermediate levels or a central depot which in turn is supplied by the spare part manufacturers. When an item fails, it is sent out to repair and a spare is plugged in. If the spare part is not available, it is backordered from the preceding repair levels. As a result, all repair levels operate according to a continuous stocking policy (S - 1, S) and the considered model intends to maximize system availability subject to a budget constraint using marginal analysis Sherbrooke, (1668). Besides, METRIC considers that the installed repair capacity is unlimited, thus the repair times and the number of components in repair are assumed to independent. Other feature of METRIC model is first-come-first-served replenishment policy at all repair level and item failure rate is assumed to follow Poisson distribution. Consequently, the number of items at bases, in transportation or in repair is approximated to be Poisson distribution. Under the Poisson distribution, the mean of backordered items are equal to their variance.

Afterwards, there have been several lines of research on enhancing METRIC outputs. One line pertains to add some features to METRIC model to tackle some practical issues. On the basis of the previous model, Muckstadt (1973) presented the MOD-METRIC to analyse two-indenture systems instead of single indenture ones. Moreover, Slay (1984) proposed VARI-METRIC model where the hypothesis of the equality of backorder mean and variance are no longer assumed. Moinzadeh et al. (1986) have delivered a decision tool to select an (S - 1, S) policy versus an (r, Q) policy. Their tool was tailored only to multi-echelon inventory systems with a single indenture. In addition, Axsater (1990) has optimized inventory base stock levels by determining average holding and shortage costs. The common characteristic of these researches is they have focus only of spar part inventory.

In all models reviewed hitherto repair capacity is assumed ample which is often an unrealistic in real-world contexts. In industrial setting of spare part inventory analysis, each repairable failed item is supplied to repair shop where reparation time encompasses generally waiting time for repair and repair time. A serious limitation of the previous models is that they work under the assumption that both waiting time and repair time are constants and independent for each component, i.e. the repair capacity is infinite. Due to budget constraints, companies invest a certain amount in repair facilities to guarantee a predefined level of maintenance performances and therefore infinite repair capacity is seldom realistic. This causes an overestimate in spare parts to maintain target availability above a predefined threshold value. Díaz et al., (1997) were the first who studied spare part management under limited repair facilities. They consider the situation where all failed items are repaired only at the central level which has a limited capacity. Their approximation for the repair time was based on queuing theory. Unfortunately, they derived model equations only for a single-server multi-class queue model due to analytical complication. Sleptchenko et al. (2002, 2005) extended the previous work by studying a more general multi-class multi-server queuing model. However, to deliver an analytical solution, they limit themselves to steady state for a given repair capacity.

Based on the above review, extensive research has been devoted to the fields of inventory location theory, queuing theory and level of repair analysis; yet research that establishes the interaction of these fields is limited. This paper reviews the current research on the spare part management and in particular, it focuses on the interaction between spare part provision and repair capacity. Its outcome will be a part of further research pertaining to provide a framework to support policy and decision-makers model that simultaneously considers a multi-echelon repair network with inventory pooling and finite repair capacity for multi-indenture systems. To our knowledge, past research has not jointly considered these three factors.

The organization of this paper is as follows. Section 2 gives the basic of VARI-METRIC technique and its mathematical model for multi-echelon and multi-indenture system. Section 3 is dedicated to reviewing the major classes of queueing theory models for finite repair capacity. Based on these models, Section 4 discusses the heuristic optimisation algorithm which takes into account the effect of limited repair capacity. This algorithm is tested in a numerical experiment in Section 5. Finally, in Section 6, the results of the study are summarised and some areas for future research are given

MODEL DESCRIPTION

The objective of our model is to jointly find out spare part stocks that satisfy installed repair capacity and desired level of system availability, while minimizing the relevant whole life costs. In particular, the purposes this model identifies spare part flow within repair network are twofold: (a) to study for a given budget threshold the required spare part inventory which maximizes system availability, and (b) to study the effect of repair capacity on maintenance performance measures (e.g., system availability and maintenance and support costs).

In this section, VARI-METRIC model for a multi-echelon repair network and multi-indenture system is described (Slay, 1984). Firstly, the model notation and hypothesis, similar as in the VARI-METRIC model, are presented. Secondly, the operational availability is expressed as a mathematical function of the requested spare part from preceding supply levels, namely backorder.

The assumption adopted in this paper is as follows:

1. All items can be repaired within repair network;
2. Failures are stationary Poisson processes and independent of the number of items under repair;
3. (s – 1, s) inventory policy is applied for all items at all repair bases.
4. The repair time of any item follows an exponential distribution.
5. There is no lateral supply, i.e., no supply or shipment between bases at the same level;

6. Each failed item is shipped between the repair bases without delay (an infinite number of transporters). The transportation time is known as order-and-ship time.
7. Backorders for different items have the same importance.
8. Repair resources are allocated to failed items according to the FCFS (first come first served) policy;
9. When repair is done, all failed items become as good as new.

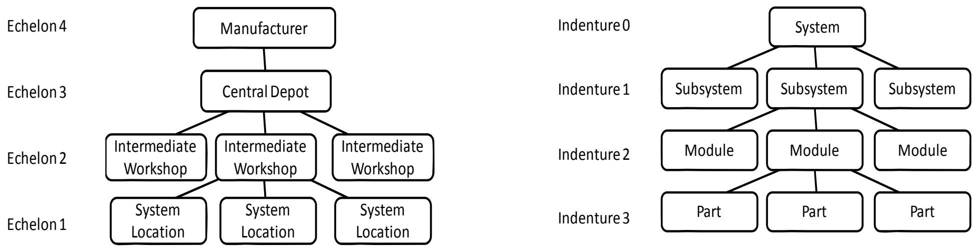

Figure 1: A multi-echelon repair network and a multi-indenture system

The notation adopted in this paper is as follows:

1. Input parameters

N	number of systems;
I	number of assemblies within each system ;
Z_{ik}	number of occurrences of assembly I in the system k;
i	= {1, 2,...,n} : set of spare parts;
j	= {1, 2,...,M} : set of local bases; the index 0 is reserved for the central depot;
λ_{ij}	failure rate of part i at base j;
S_{ij}	stock level for part i at base j (decision variable);
R_{ij}	probability that item i can be repaired at base j;
Rt_{ij}	mean repair time of part i at base j;
O_{ij}	mean transportation time of item i between the base j and its supplier (referred also by order-and-ship time)
c_i	price of item i
$BO_{ij}(S_{ij})$	numbers of backorders for item i at base j as function of the stock levels S_{ij};
$PBO_{ij}(S_{ij})$	backorder probability for item i at base j as function of the stock levels S_{ij};

2. decision parameters

N	number of systems;
I	number of assemblies within each system ;
Zik	number of occurrences of assembly I in the system k ;

3. intermediate parameters

N	number of systems;
I	number of assemblies within each system;

The number of backorders, denoted by $BO_{ij}(S_{ij})$, stands for requested quantities of item i from its supplying location j. $BO_{ij}(S_{ij})$ is the positive value representing the difference between the needed spare parts of item i, denoted by pipeline P_{ij}, and the stock at hand S_{ij} at the base j; $BO_{ij}(S_{ij}) = (P_{ij} - S_{ij})^+ = \max(0, P_{ij} - S_{ij})$. In other word, if the stock at hand S_{ij} is greater than P_{ij} there will be no spare part demand from the preceding repair bases.

In order to be consistent with spare part literature and its pioneer model METRIC, the Number of Backorders BO is considered as the measure of system performance. Employing Sherbrooke's approximation, the average system availability of system is:

$$A = \prod_{i=1}^{N}\left(1 - \frac{EBO_i(S_i)}{Z_i}\right)^{Z_i} \qquad (1)$$

With

Z_i	number of occurrences of part i in system

N	number of items i
$\mathrm{EBO_i(S_i)}$	expected backorders of item type i for stock level Si $(0 \le \mathrm{EBO_i(S_i)} \le N * Z_i)$

The difference $(1-\frac{\mathrm{EBO_i(S_i)}}{Z_i})$ represents the availability of item i. This term to the power Z_i represent the availability of a system due to item i. Finally, multiplying over all service items (i=1..N) gives the general expression for the availability of a system as a result of the stocking policy for service items. Therefore, the probability that the system is not down due to a lack of an item i is $(1-\frac{\mathrm{BO_i(S_i)}}{Z_i})^{Z_i}$. The above Sherbrooke's formula assumes that the probabilities for different item i are independent and the system is a serial structure in reliability terminology. By applying logarithm to A and taking the expectation of (1), the average availability of all systems at downstream repair base ech(N) is:

$$A = 1 - \frac{1}{\mathrm{ech(N)}} \sum_{i=1}^{\mathrm{ech(N)}} \sum_{j=1}^{\mathrm{ind(1)}} \mathrm{BO_{ij}}\,(S_{ij}) \qquad (2)$$

The spare part management objective is to determine inventory policies at bases to minimize spare holding costs while maintaining an average availability greater than a given threshold value. Sherbrooke shows that maximising this availability function is approximately equivalent to minimising the sum of the expected backorders. Consequently, the optimization of spare part inventory will be:

$$\begin{cases} \min \sum_{i=1}^{\mathrm{ech(N)}} \sum_{j=1}^{\mathrm{ind(1)}} \mathrm{BO_{ij}}\,(S_{ij}) \\ \text{Subjet to} \\ S_{ij} \ge 0 \\ \sum_{i=1}^{n} c_i \sum_{j=1}^{\mathrm{ech(N)}} S_{ij} \le \text{Budget} \end{cases} \qquad (3)$$

The above mentioned integer programming model requires the identification of steady-state expressions for the backorder and stock levels. Let us consider the following situation to derive backorder expressions. In the case where there are plenty of spares at bases to satisfy demand ($S_{ij} \sim \infty$), there will be no delay. However, at low spare quantities there will be delay time for transportation time (order and receive between bases) plus repair time. Therefore, delay can be expressed as a function of stock level S; if the demand x is less than S there will be no delay but if there are greater than S, then the supply of (x-S) items will be delayed. The expected number of delayed items or the expected number of backorder may be expressed:

$$\mathrm{BO_i(i,j)} = \sum_{x\,=\,S+1}^{\infty} (x - S) * P(x > S) \qquad (4)$$

As a result, for each stock level S the expected backorders is obtained as a function of the stock level S, the demand x (pipeline) and the pipeline distribution probability P(x>S).

The failure rate $\lambda_{i\,j}$ of item i at base j is computed by adding the following two values:

1. The failure rates of this item at downstream bases at which repair actions could not be done $\sum_{\mathrm{base}\ l>j} \lambda_{i\,l} * (1 - r_{i\,1})$: where $r_{i,j}$ is the probability that an item i could be repaired at base j.
2. The failure rates of higher indenture items :$\sum_{k=1}^{\mathrm{parent}\ i} q_{k\,i} * \lambda_{k\,l} * r_{k\,j}$ where : $q_{k\,i}$is the probability that item k is the cause of the failure of its parent i and
   $r_{k\,j}$: the probability that an item k could be repaired at base j.

Thus, the failure rate of any item i will be:

$$\lambda_{i\,j} = \sum_{\mathrm{base}\ l>j} \lambda_{i\,l} * (1 - r_{i\,1}) + \sum_{k=1}^{\mathrm{parent}\ i} q_{k\,i} * \lambda_{k\,j} * r_{k\,j} \qquad (5)$$

Starting by the highest indenture items, all failure rates can be calculated recursively. the demand quantities or pipeline for the bases are computed according to METRIC assumptions. That is, the repair time and order and ship time from the higher bases are independent and both follow Poisson distribution with parameters $\lambda_{i\,j} * rt_{i\,j}$ and $\lambda_{i\,j} * O_{i\,j}$. The pipeline of item i at base j will be therefore the superposition of the mean of these two Poisson distributions multiplied by respective probabilities. Thus:

$$P_{i\,j} = \lambda_{i\,j} * rt_{i\,j} * r_{i\,j} + \lambda_{i\,j} * O_{i\,j} * (1 - r_{i\,j}) \qquad (6)$$

From Equation 6, it is easy to notice that the pipelines should be extended to take into account pipeline from both higher bases and higher indentures. Only a fraction of the pipeline at base j suppliers originates from base j. As considered in the literature, orders are filed in First Come First Served basis. Consequently every order has a probability $f_{i\,j} = \frac{\lambda_{i\,j} * (1 - r_{i\,j})}{\lambda_{i\,sup(j)}}$ to originate from base j.

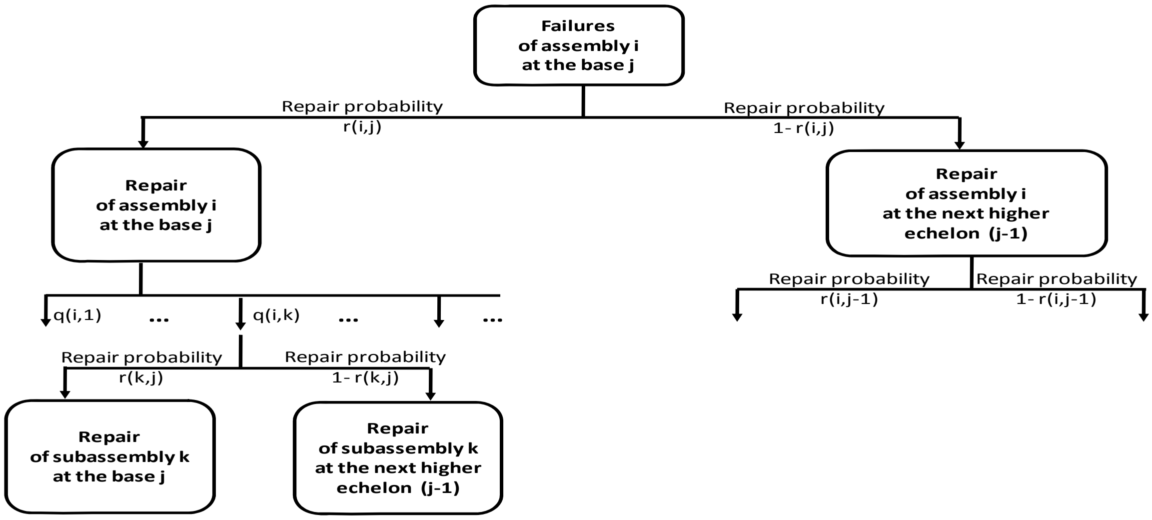

Figure 2: A multi-echelon repair network process

Then the number of orders that stems from base j, equals $f_{i\,j} * BO_{i\,sup(j)}$. Pipeline expression generated from higher indentures is derived as follow: Let us consider an item k for which j is a parent. Only a fraction $h_{i\,j\,k} = \frac{r_{k\,j} * \lambda_{i\,j} * q_{i\,k}}{\lambda_{k\,i}}$ of the backorders for item k at location j arising from item j. then, the mean value of pipeline generated from higher indenture of item j equals : $\sum_{k \in SA(j)} (h_{i\,j\,k} * BO_{k\,j})$. Hence, the pipeline of item i at base j can be written as:

$$P_{i\,j} = \lambda_{i\,j} * rt_{i\,j} * r_{i\,j} + \lambda_{i\,j} * O_{i\,j} * (1 - r_{i\,j}) + f_{i\,j} * BO_{i\,sup(j)} + \sum_{k \in SA(j)} (h_{i\,j\,k} * BO_{k\,j}) \quad (7)$$

The equation 7 may be interpreted as follows. The term $\lambda_{i\,j} * O_{i\,j} * (1 - r_{i\,j})$ represents the part of pipeline due to the transportation process between bases; the terms $\lambda_{i\,j} * rt_{i\,j} * r_{i\,j}$ and $f_{i\,j} * BO_{i\,sup(j)}$ denote the part that is delayed due to a lack of stock at base j and its supplier echelons and finally the term $\sum_{k \in SA(j)} (h_{i\,j\,k} * BO_{k\,j})$ refers also to the lack of stock of higher enclosure indentures at base j. From equations 4 and 7, we noticed that the expected backorder is computed from pipeline values and the later are calculated from expected backorder values. As result, backorders are computed recursively.

For a given base stock S, evaluation of the steady state backorder probabilities can be done as described in , by fitting discrete distribution on the first two backorder moments, e. i expected value and variance. In METRIC it is assumed that the variance equals the expected backorder of items in repair process, however several researchers have noticed that variance to mean ratio is no longer equals to 1 such under Poisson distribution, but it is usually greater than 1 in practice. Slay and Graves developed an approximation for backorder probabilities by applying binomial distribution and the negative binomial distribution respectively. In this paper, the approximation is obtained by Poisson, Negative Binomial or geometric distributions as described by Adan et al. Similar to the expression for the expected backorders, the variance equals to ():

$$Var_{i\,j} = \lambda_{i\,j} * rt_{i\,j} * r_{i\,j} + \lambda_{i\,j} * O_{i\,j} * (1 - r_{i\,j}) + f_{i\,j} * (1 - f_{i\,j}) * BO_{i\,sup(j)}$$

$$+ f_{i\,j}^2 * var(BO_{i\,sup(j)}) + \sum_{k \in SA(j)} (h_{i\,j\,k} * (1 - h_{i\,j\,k}) * BO_{k\,j} + h_{i\,j\,k}^2 * var(BO_{k\,j})) \quad (8)$$

Finally, expression of the Expected Backorders (EBO) as the measure of system performance is given by the following equations:

$$BO_{i\,j} = \sum_{x\,=\,S+1}^{\infty} (x - S) * P(x > S) = \sum_{x=0}^{\infty} (x - S) * P(x > S) - \sum_{x=0}^{S} (x - S) * P(x > S)$$

$$= \sum_{x=0}^{\infty} x * P(x > S) - S - \sum_{x=0}^{S} (x - S) * P(x > S) \qquad = P_{i\,j} - S - \sum_{x=0}^{S} (x - S) * P(x > S) \quad (9)$$

This is an explicit part of the objective function which seeks to minimize the sum of expected backorders at the downstream bases. The equation 9 shows that $BO_{i\,j}$ decreases whenever there is an increase of stock level S of item i. Therefore, the problem 3 may be solved by using a greedy heuristic method according to the following steps. First, an initial base stock level is set for each item i at all bases. The corresponding Expected backorders and investment cost

C are computed. Since increase of S leads to a decrease of BO, the reduction in sum of expected backorder per invested dollar is calculated when only Si is increased by one at base j. This sum expected backorder reduction per invested dollar is denoted by the Δ_{ij}.

$$\Delta_{ij} = \frac{\sum_i \sum_j BO_{ij}(S) - \sum_i \sum_j BO_{ij}(S+e_{ij})}{c_i} \qquad (10)$$

Where: e_{ij} is a matrix with all elements equal to zero, except for element i,j which is equal to 1.

The increase by one of item i at base j leading to the maximum Δ_{ij} is selected for stock replenishment. In addition, this replenishment will increase the holding stock cost C by ci. This procedure is carried out until the budget is reached.

FINITE REPAIR CAPACITY

The underlying assumption in the above model is that repair capacity is infinite and as a result, the repair shops are not considered as a decision variable. Díaz, et al. (1997) first relax this assumption by considering limited repair facilities only at the central base. Other researchers tried to extend the METRIC method to study the impact of finite capacity, Aboud (1996), Sleptchenko et al., (2002), Kim et al. (2000). They have shown that limited capacity has a considerable effect on system performance for a single indenture and one or two-echelon repair network.

On the other hand, queuing theory has been the solution for range of practical problems in telecommunication, manufacturing and computer systems. Then, it is obvious that the more suitable way to manage spare parts considering also queuing network approach. There is an extensive literature on queuing theory (). The M/G/K queuing system is one of the most used models for multi-server systems. The symbol M means that the jobs arrive according to a Poisson process with rate λ ; the symbol G assumes that service time is independent and identically distributed random variables having a general distribution and K refers to the number of identical servers working with a First-Come-First-Serve (FCFS) policy. Any job received immediately service only when a free server exists, otherwise it waits in the FCFS queue.

In practice, repair shops are generally run by a limited quantity of equipment and multi-skilled crew that are able to handle at the same time a certain number of repair jobs. This gives rise to multi-server configuration, where failed items arriving with Poisson process are either in the queue or in service. Therefore, failed items in repair shops are modelled using M/G/K queuing theory.

The mean and variance of the number of items in the repair shops are given by the following approximations based on (Whitt, 1983, 1993):

$$E(N) = \lambda * \left[\left(\frac{1+C^2}{2}\right)\left(\frac{p_0}{k*\mu}\frac{(k*\rho)^k}{(1-\rho)^2*k!}\right) + \frac{1}{\mu}\right] \qquad (11)$$

$$V(N) = E(N^2) - E(N)^2 \qquad (12)$$

Where:

$$E(N^2) \approx E\left(N^2_{M/M/K}\right) * \frac{E(N)^2}{E(N_{M/M/K})^2} \qquad (13)$$

$$E\left(N_{M/M/K}\right) = k*\rho + \frac{\rho*(k*\rho)^k}{(1-\rho)^2*k!}\ p_0 \qquad (14)$$

$$E\left(N^2_{M/M/K}\right) = k*\rho*\left(1+\frac{(k*\rho)^k}{(1-\rho)*k!}\ p_0\right) + \frac{\frac{(k*\rho)^k}{(1-\rho)*k!}\ p_0\left[1+\rho*\left(1-\frac{(k*\rho)^k}{(1-\rho)*k!}\ p_0\right)\right]}{(1-\rho)^2} + E(N_{M/M/K})^2 \qquad (15)$$

Where:

$$p_0 = \left[\sum_{j=0}^{k-1}\frac{(k*\rho)^j}{j!} + \frac{(k*\rho)^k}{(1-\rho)*k!}\right]^{-1}$$

k	number of servers at the repair shop,
μ	service rate of each server,
λ_i	arrival rate of failed item i,
$\lambda = \sum \lambda_i$	arrival rate at the repair shop,
$\rho = \frac{\lambda}{k*\mu}$	utilization of the repair shop,
S	service time at the repair shop, $E(S) = \frac{1}{\mu}$
N	number of items at the repair shop,
Q	number of items in queue at the repair shop,

W waiting queue time at the repair facility,

C, coefficient of variation for random variable $C = \frac{\text{Varaince}}{\text{mean}^2}$

P_n probability that there are n items at the repair shop.

These first two moments concern only items under repair service, however, the repair time includes as well the waiting time in the queue when servers are full. The waiting time is, in turn, presented by another random variable $Q^+ = \mathrm{Q/Q} > 0$ (the conditional queue length given that the queue is not empty). Its mean and variance are given by:

$$E(Q^+) = \mathrm{E(Q)/p(Q > 0)} = \left[\mathrm{E(N)} - \frac{\lambda}{\mu}\right]/\mathrm{p(Q > 0)} \qquad (16)$$

Where :

$$\mathrm{p(Q > 0)} \approx \rho * \mathrm{p(W > 0)} = \rho * \min(\pi, 1)$$

$$\pi = \rho^2 * \pi_a + (1 - \rho) * \pi_b$$

$$\pi_a = \min\left\{1, \frac{1-\Phi\left(\frac{(1+C_s^2)*(1-\rho)\sqrt{k}}{C_a^2+C_s^2}\right)}{1-\Phi\left((1-\rho)\sqrt{k}\right)} \mathrm{p}\left(\mathrm{W_{M/M/K}} > 0\right)\right\} = \mathrm{p}\left(\mathrm{W_{M/M/K}} > 0\right)$$

$$\pi_b = \min\left\{1, \frac{1-\Phi\left(\frac{2(1-\rho)\sqrt{k}}{1+C_s^2}\right)}{1-\Phi\left((1-\rho)\sqrt{k}\right)} \mathrm{p}\left(\mathrm{W_{M/M/K}} > 0\right)\right\}$$

$$\mathrm{p}\left(\mathrm{W_{M/M/K}} > 0\right) = \rho$$

$\Phi(\dots)$ is a cumulative function of standard normal distribution

The term π_a is equal to $\mathrm{p}\left(\mathrm{W_{M/M/K}} > 0\right) = \rho$ since arrival time is assumed to be a Poisson process for which mean^2 = variance = λ^2 and the coefficient of variation $C = \frac{\text{Varaince}}{\text{mean}^2} = 1$.

The variance of waiting time Q can be obtained by computing its coefficient of variation C_{Q^+} :

$$\begin{cases} C_{Q^+}^2 = \frac{1}{E(Q^+)} - \frac{\mathrm{p(Q>0)}}{\mathrm{p(W>0)}}(C_D^2 + 1) \\ C_D^2 = 2 * \rho - 1 + 4 * (1 - \rho)\frac{d_s^3}{3*(C_s^2+1)^2} \\ d_s^3 = \begin{cases} 3 * C_s^2 * (C_s^2 + 1) & if\ C_s^2 > 1 \\ (2C_s^2 + 1) * (C_s^2 + 1) & if\ C_s^2 < 1 \end{cases} \end{cases} \qquad (17)$$

Finally, backorders given by equation (9) can be approximated based on the first two moments of the numbers of items in the pipeline. The common technique to obtain this is set by Adan et al. (1996). Based on this approximate, the probability distribution for the pipeline P(X>0) is fitted on the first two moments of negative binomial, Poisson or mixed two geometric distribution.

THE ALGORITHM FOR CALCULATING OPTIMUM SPARE PART INVENTORY

On the basis of the mathematical expressions (10), the optimization algorithm has been defined as the maximization of the quotient of the backorders BO to cost increment, i.e., $\Delta_{ij} = \frac{\sum_i \sum_j BO_{ij}(S) - \sum_i \sum_j BO_{ij}(S+e_{ij})}{c_i}$. This criterion function is followed during each iteration step made for identifying the spare part which should be added to the stock. The constraint is that the total cost of spare does not exceed the allowed budget.

The algorithm provides efficient solutions $S_{1,0}$, $S_{2,0}$, $S_{3,0}$, …, $S_{i,j}$, … at all repair shops and for all system enclosed items. Throughout the algorithm $S_{i,j}$, denotes generated efficient solution item i at the echelon j, C($S_{i,j}$,) stands for the corresponding spare part cost and BO($S_{i,j}$,) refers to the corresponding expected number of backorders. The algorithm ends when there is no longer any efficient solution with C ≤ budget. The stock allocation is obtained by using the following iteration algorithm:

- Step 0: since the optimisation procedure of the problem (3) is a greedy heuristic, a prerequisite of this procedure is the function backorder BO against the cost C should be convex. Rustenburg et al, (2002) have examined the effect of initial stock on the curve convexity and they found that this stock should be set equal to $S_{i\,j} =$

$round\left(\lambda_{ij} * rt_{ij} * r_{ij} + \lambda_{ij} * O_{ij} * (1 - r_{ij})\right)$ and $S_{i0} = round(\lambda_{i0} * rt_{i0} * r_{i0} + \lambda_{i0} * O_{i0} * (1 - r_{i0}))$ at the depot base.

- Step 1: Stock level $S_{i,j}$ is increased by 1 for all i and all j.
- Step 2: the expected numbers of backorders BO($S_{i,j}$) are calculated.
- Step 2: The mean and the variance of waiting time and service at repair shop are calculated.
- Step 3: The mean and the variance of pipeline value are calculated.
- Step 4: Fit a discrete distribution to mean and variance of pipelines assuming that their constituents are uncorrelated.
- Step 5: the expected numbers of backorders BO($S_{i,j}$) are calculated.
- Step 6: the quotients Δ_{ij} is calculated
- Step 7: the pair (i,j) leading to the highest value of Δ_{ij} is selected.
- Step 8: Stock level $S_{i,j}$ is increased by 1 for the selected (i,j)
- Step 9: if the criterion stock cost C ≤ budget is satisfied then go to step 1, otherwise stop.

For each generated solution $S_{i,j}$ is different from the previously generated solution in just one component. $\Delta = \frac{\Delta BO}{\Delta C} = \frac{\text{decrease in } BO(S_{i,j}) \text{ if } S_{i,j} \text{ is increased by 1}}{\text{increase in } C(S_{i,j}) \text{ if } S_{i,j} \text{ is increased by 1}}$. Therefore, in each of the above steps, the increase the stock $S_{i,j}$ by 1 should generate marginally the largest decrease of BO(s) per invested dollar.

COMPUTATIONAL EXPERIENCE

The main purpose of our experiments is to obtain a curve for availability and spare pooling cost. The proposed algorithm is written in Matlab and the experiments are performed on a Pentium (…..) compatible PC system. The data for the experiment concerns gas turbines depicted in the figure 4. This system is installed either in boosting gas stations or in some gas power plant and will be only considered by element structure presented in figure 3. The equipment is divided into six main subsystems: trunnion, air inlet, compressor, combustion, turbine and turning gear subsystem. Those subsystems have their enclosure components performing specific tasks in connection with the subsystem main function.

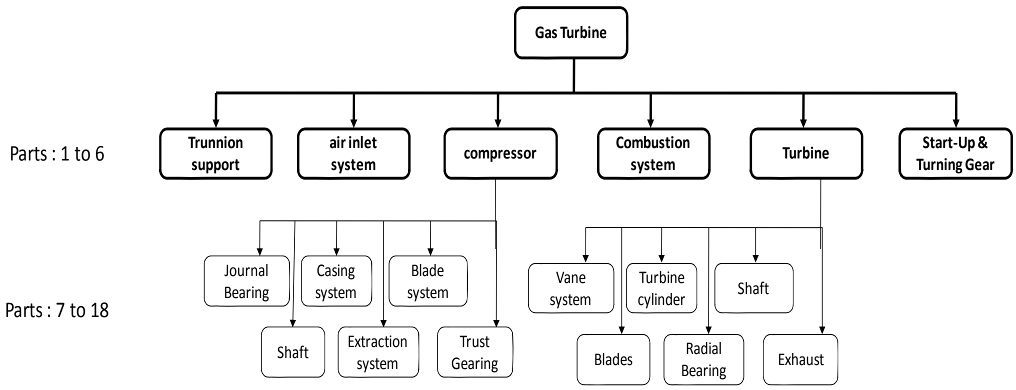

Figure 3 : Gas Turbine Element Breakdown Structure

Hence the total number of pare under study is 18 parts. We first consider the two examples with infinite and limited repair facilities. The input data for these examples are shown in Table 1. The different service times we considered was based on the average response time, defined as the average time it takes to repair or to receive a spare part after a failure is reported.

Table 1 : Input data (λ is given in failures per year; Rt, and Ot in years)

Item	Part1	Part2	Part3	Part4	Part5	Part6	Part7	Part8	Part9	Part10	Part11	Part12	Part13	Part14	Part15	Part16	Part17	Part18
λ	8	9	5	4	4	7												
repair probability R	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Repair time Rt	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
order & ship time Ot	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
q	0	0	0	0	0	0	0.17	0.34	0.10	0.21	0.09	0.10	0.20	0.20	0.09	0.21	0.10	0.20
part cost ci	298	298	589	486	884	283	282	273	267	266	249	241	240	231	229	218	216	205

First of all, the greedy approach optimization as described above has been applied by considering infinite repair capacity and the problem solution was obtained until the availability was reached 99.99%. In this example, 666 possible solutions have been examined and the final result when inventory cost attains the budget limit is shown in the table 2. Besides, all these possible solution represent the optimal pairs (inventory cost C, system availability A) for which any invested dollar have led to the maximum increase in system availability. These pairs constitute a so-called spare part investment versus availability curve, are graphically depicted in Fig. 5. For example, the figures in the table 2 correspond to the optimal provision of spare part within repair shops (in this case there 6 bases to be supplied) when system availability attains 99.99%.

Table 2: Spare part allocation throughout repair network

Item	Part1	Part2	Part3	Part4	Part5	Part6	Part7	Part8	Part9	Part10	Part11	Part12	Part13	Part14	Part15	Part16	Part17	Part18
base	3	4	2	2	2	3	1	1	1	1	1	1	1	1	1	1	1	1
loc1	7	7	5	5	5	6	1	1	1	1	1	1	1	1	1	1	1	1
loc2	7	7	5	5	5	6	1	1	1	1	1	1	1	1	1	1	1	1
loc3	7	7	5	5	5	6	1	1	1	1	1	1	1	1	1	1	1	1
loc4	7	7	5	5	5	6	1	1	1	1	1	1	1	1	1	1	1	1
loc5	7	7	5	5	5	6	1	1	1	1	1	1	1	1	1	1	1	1

Once the provision problem has been solved, the experiments were performed to evaluate the effect of various repair shop capacities on both system availability and inventory costs. From the fig. 5, it is clear that additional investment in repair capacity (increasing the number of repair servers) results in a decrease in the inventory cost for the same availability values. In the case of infinite repair capacity, the blue line represents the asymptote from which no further reduction in inventory cost will achieve by increase in repair capacity. Besides, this asymptote is almost identical to that obtained with 4 repair servers. Therefore, there is not worth to envisage more than 4 repair servers per repair shop.

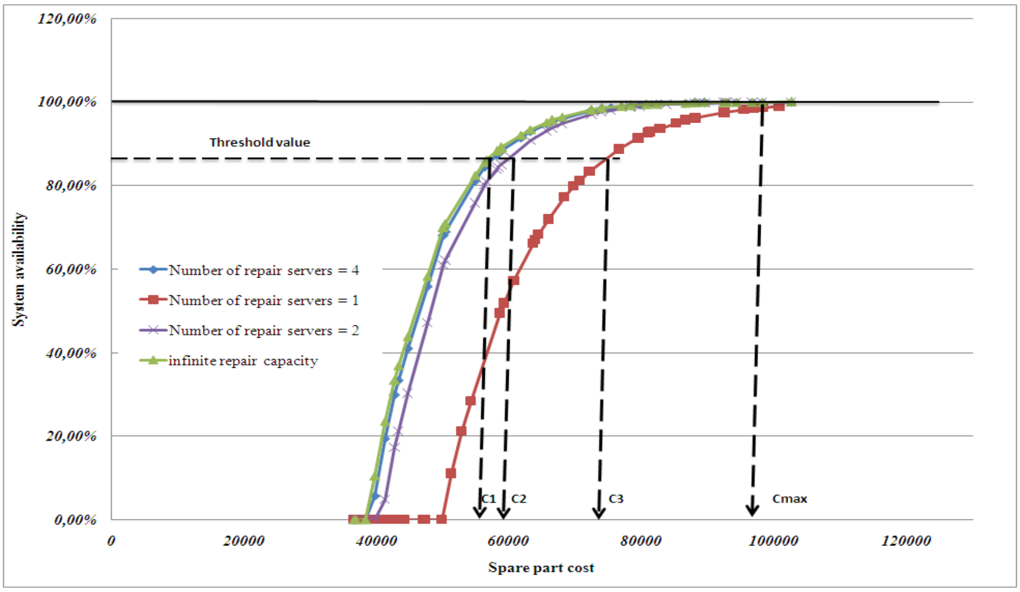

Figure 4 : System availability vs spare part costs.

For a given availability values, asset managers can decide whatever to repair failed items or to possess enough spares for immediate replacements. For instance, the difference inventory investment (C3-C1) for the same availability may increase to 27%. Alternatively, the benefit in inventory investment is approximately 3000 when there are more additional repair servers. Therefore, the question is: given the availability threshold value, what is the cost effective investment decision in reapir capacity vs. inventory? The answer is displayed in the table 6 in, which the differences in spare part costs for an equal availability.

CONCLUSION AND FUTURE WORKS

This study highlights the advantage that maintenance efficiency could be achieved with the adoption of integrated logistics support elements. More specifically, in spare parts management and in case identical equipment can be installed in different geographical areas requiring very close repair services. A multi-echelon repair network is considered in this paper that includes a central depot, and many field bases. The results discussed show the impact of spare part modelling on the desired system availability. It was demonstrated that the queuing theory could provide an opportunity to better estimate the required spare parts and especially if the repair shops have a limit capacity. The study also reveals the trade-off between the spares inventory and investment in repair facilities. In an intensive system industry like petroleum industry, it may be a worthwhile policy to reduce the capital investment in repair spare by expediting engine overhauls by putting more manpower on the jobs. Such a policy could in turn provide more employment to the people.

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