ENERGY EQUATION IN 1D GAS PIPELINE FLOW – EFFECT OF TURBULENT DISSIPATION

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1. ABSTRACT

Transportation of natural gas through high pressure transmission pipelines can be modeled by numerically solving the partial differential equations for mass, momentum and energy conservation for one-dimensional compressible viscous heat conducting flow. Since the one-dimensional version is a result of averages over the pipe cross-section, and the flow normally is turbulent, the order of averaging in space and time is an issue, in particular for the dissipation term. The Reynolds decomposition and time averaging should be performed first, followed by the contraction to the one-dimensional version after the cross-sectional averaging. The result is a correction factor, close to unity, on the usual expression of the dissipation term in the energy equation. This factor will affect the temperature distribution along the pipeline. The effect is most dominant for rough pipes (surface roughness of 10 μm or more) operating at high Reynolds numbers (10^7 or higher). If the pipeline is also thermally isolated such that the flow can be considered as adiabatic, then the effect of turbulent dissipation is even larger. For smooth pipes with a roughness of less than 10 μm the effect of turbulent dissipation is so small that it can in most cases be neglected. Simulations were validated against operational data available from high pressure natural gas pipelines. It was shown that correcting the dissipation term in the energy equation due to turbulent flow gives slightly more accurate values for the modeled temperature.

Key words: Natural gas pipelines, numerical simulations of one-dimensional flows, temperature effects

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2. INTRODUCTION

Gassco is a state owned Norwegian company responsible for the operation of 7800 km of gas pipelines transporting natural gas from the Norwegian continental shelf to continental Europe and the United Kingdom, covering around 15% of the European natural gas consumption. Natural gas is transported from the continental shelf through high pressure offshore transmission pipelines on the seabed to processing terminals on mainland. The lengths of the pipelines lie in the range 100 – 1000 km, the longest being close to 1200 km (Langeled). Pressure transmitters, flow meters and temperature measurements are only located at the inlet and outlet of the pipelines. To know the state of the gas between these two points one has to rely on computer models. These models are used for general monitoring of the gas and predicting the pipeline hydraulic capacity. To optimize the utilization of the pipelines it is crucial that the transport capacity is estimated as accurately as possible. During the winter the demands for natural gas usually exceeds the estimated transport capacity of the pipeline network. More accurate modeling of the flow can lead to improved use of the pipeline network capacity, while at the same time allow for safe operation.

Since 2004 the R&D Institute Polytec has been actively working together with Gassco to improve flow modeling. Areas of research have been improved friction factor correlation [1], viscosity measurements and implement new viscosity correlations [2] and more accurate modeling of heat transfer from the pipelines to the surroundings [3]. In recent years collaboration with the Norwegian University of Science and Technology has been established to do research on transient flows and temperature modeling. This paper will focus on work related to temperature modeling in high pressure transmission pipelines. Correct temperature modeling is very important as the temperature is an important parameter when determining the pipeline hydraulic capacity. Modeling the correct temperature is quite complex, because there are several terms is the energy equation one has to model in approximate ways. These include change in enthalpy, Joule-Thomson effect, dissipation term and heat exchange with the surroundings. This paper will demonstrate how the dissipation term should be represented in turbulent pipeline flow.

3. THEORY

3.1 Governing equations

The simulation of natural gas transmission in pipelines involves the numerical solution of a system of initial valued partial differential equations for mass, momentum and energy conservation. The basic equations for one-dimensional, unsteady non-isothermal compressible flow can be found in base literature articles such as the one by Thorley and Tiley [4]. The governing equations are

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \tag{1.1}$$

Momentum

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = -\frac{W}{A} - \rho g \sin\theta \tag{1.2}$$

Energy

$$\rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial x} - \frac{\partial p}{\partial t} - u \frac{\partial p}{\partial x} = \frac{Wu + \Omega}{A} \tag{1.3}$$

W is the frictional force per unit length of the pipe

$$W = \frac{A\rho f u\,|\,u\,|}{2D} \tag{1.4}$$

and Ω is the heat flow into the pipe per unit length and unit time

$$\Omega = \frac{4UA(T - T_a)}{D} \tag{1.5}$$

The density ρ may be traded for the pressure p by using a real gas equation of state

$$\frac{p}{\rho} = ZRT \tag{1.6}$$

where Z = Z(p,T) is the compressibility factor. Chaczykowski [5] gives a review of and compares different equations of state commonly used in the gas industry. In this paper the Soave-Redlich-Kwong (SRK) equation of state has been used, which allows for convenient compressibility factor calculations.

The friction factor f depends on the Reynolds number of the flow and the surface roughness ε. There are several different correlations for the friction factor. One of the most widely used is the Colebrook-White correlation

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{2.51}{\mathrm{Re}\sqrt{f}} + \frac{\varepsilon}{3.71D}\right] \tag{1.7}$$

3.2 Energy equation – 3D to 1D transformation

Equations 1.1 - 1.3 are found by integrating the three-dimensional equations for mass, momentum and energy across the pipeline cross-section. The equations which are used for this transformation are the ones which are valid for laminar flow, the only difference being the substitution of the friction factor from laminar to turbulent f. Natural gas flow in large diameter pipelines such as the ones operated by Gassco usually has a Reynolds number of the order of magnitude 10^7, which corresponds to turbulent flow. A turbulent velocity profile should therefore be considered when going from a three-dimensional to a one-dimensional version of the governing equations. For the energy equation, the enthalpy version for three-dimensional flow in Cartesian axis is

$$\rho\frac{Dh}{Dt} - \frac{Dp}{Dt} = \tau_{ij}\frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right) \tag{1.8}$$

The first term on the right hand side is the dissipation term, often denoted Φ. Dissipation is the breakdown of mechanical energy to thermal energy due to viscous forces in the fluid. To evaluate Equation 1.8 for turbulent flow, the usual Reynolds decomposition is made first with physical properties written as the sum of a mean and fluctuating (turbulent) part

$$h = \bar{h} + h' \tag{1.9}$$

$$u_i = \overline{u_i} + u_i' \tag{1.10}$$

$$p = \overline{p} + p' \tag{1.11}$$

Equations 1.9 – 1.11 are inserted into equation 1.8 and averaged over time. The dissipation term is then developed into [6]

$$\overline{\Phi} = \overline{\tau_{ij} \frac{\partial u_i}{\partial x_j}} \rightarrow \left(\overline{\tau_{ij}} - \rho \overline{u_i' u_j'}\right) \frac{\partial \overline{u_i}}{\partial x_j} \rightarrow \tau_{ij}^{tot} \frac{\partial \overline{u_i}}{\partial x_j} \tag{1.12}$$

where $\rho \overline{u_i' u_j'}$ are the turbulent Reynolds stresses. To take Equation 1.8 into the one-dimensional version that is usually required for pipe flow modeling, a next average, over the cross section of the pipe is made. This is trivial for all terms, except the one for dissipation

$$< \overline{\Phi} >= \frac{1}{A} \int \tau \frac{\partial \overline{u}}{\partial y} dA \tag{1.13}$$

To perform the areal integration, an expression for the turbulent velocity profile is needed. The universal turbulent velocity profile can be divided into three regions as suggested by Prandtl and von Karman [7]. These regions are the viscous sublayer, logarithmic overlap layer and the core layer. Using classical expressions for the velocity profile and shear stresses in these regions, we end up with the following expression for the dissipation term [8]

$$< \overline{\Phi} >= \rho \frac{fu^3}{2D} \left(\frac{C_f}{2}\right)^{1/2} \left[y_0^+ + \frac{1}{\kappa} \ln \frac{R^+}{y_0^+} - \frac{3}{2\kappa} + \frac{y_0^+}{R^+ \kappa} \right] \tag{1.14}$$

where C_f is the Fanning skin friction, κ the von Karman constant and, y_0^+ , R^+ are dimensionless sublayer thickness and pipe radius respectively.

3.3 Turbulent dissipation factor

The dissipation term as written in Equation 1.3 is a simple extrapolation from laminar flow by shifting the friction factor from 64/Re to f in Equation 1.7

$$< \overline{\Phi} >= \frac{Wu}{A} = \rho \frac{fu^3}{2D} \tag{1.15}$$

But when turbulent structure is included, the dissipation term should be (as demonstrated in the previous section)

$$< \overline{\Phi} >= \rho \frac{fu^3}{2D} F(\mathrm{Re}, \varepsilon) \tag{1.16}$$

where the correction factor F follows from Equation 1.14

$$F = \left(\frac{C_f}{2}\right)^{1/2}\left[y_0^+ + \frac{1}{\kappa}\ln\frac{R^+}{y_0^+} - \frac{3}{2\kappa} + \frac{y_0^+}{R^+\kappa}\right] \tag{1.17}$$

where $C_f = f/4$, $R^+ = \mathrm{Re}*(C_f/2)^{1/2}/2$ and y_0^+ is set to 10 (according to Figure 6-11, page 420 White [6]). The value of F for different Reynolds numbers and typical pipe roughness values is shown in Figure 1.

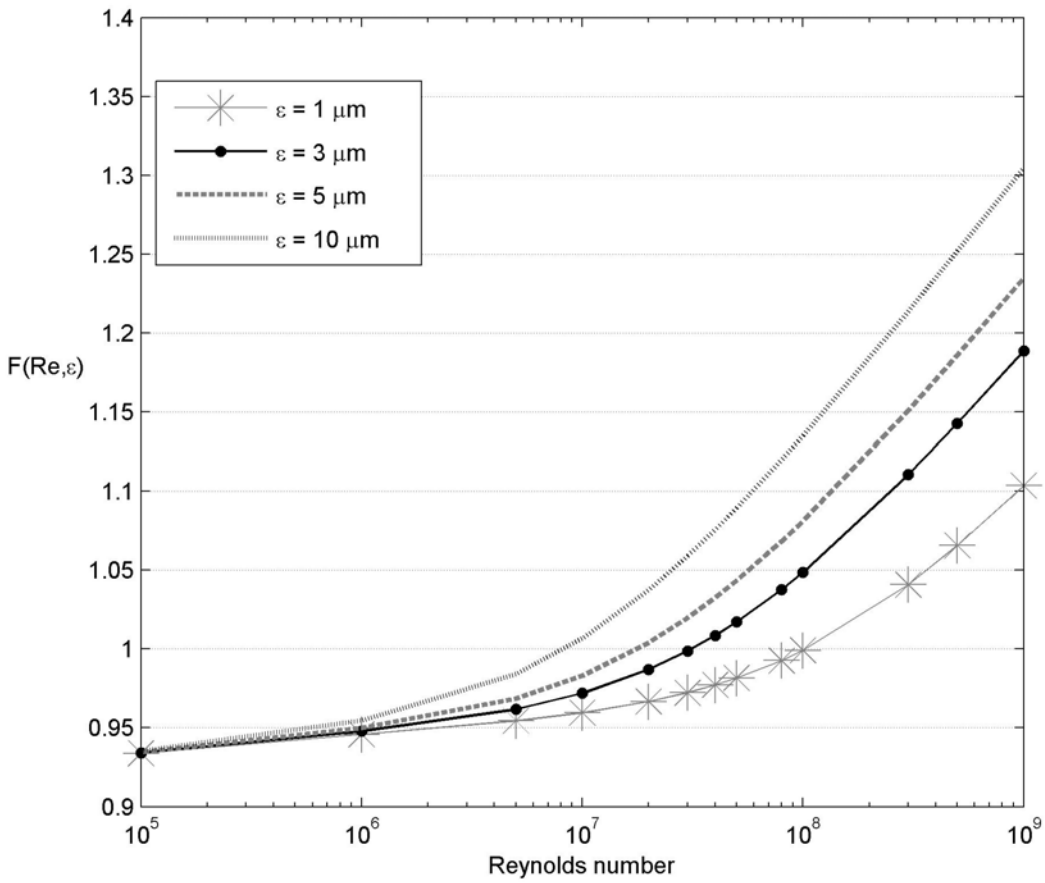

Figure 1: Correction factor F as a function of Reynolds number and pipe roughness. Results are shown for roughness values 1, 3, 5 and 10 µm. Values for F have been found using Equation 1.17.

As can be seen from Figure 1, the correction factor F due to turbulent flow is close to 1 under typical operating conditions for natural gas transport, which corresponds to Reynolds numbers of 10^7 and pipe roughness of 1 – 10 µm. Depending on the Reynolds number of the flow and the roughness of the pipe the correction factor typically lies in the range 0.95 < F < 1.05. As pointed out by Chaczykowski [9], all the terms in the energy equation are of the same order of magnitude, and even though a correction of ± 5% is small, it is none the less important for predicting an accurate value of both the outlet temperature and pressure. If the system is adiabatic with no heat exchange between the pipeline and the surroundings, the last term in Equation 1.8 would be equal to zero, and the only term left on the right hand side is the dissipation term. Adiabatic flow corresponds to rapid transients where fast dynamic changes in the pipeline parameters do not allow heat transfer to take place

between the gas and the pipeline surroundings. An accurate prediction of the dissipation term is therefore desirable, under such conditions.

3.4 Numerical scheme

A fully implicit finite difference method is used to solve the governing equations. In Equations 1.1 – 1.3 the density is replaced by the pressure by using the real gas equation of state and the enthalpy h can be obtained in terms of p, Z and T by using thermodynamic identities (see Abbaspour and Chapman [10]). The pipeline is divided into segments of equal length, and for each segment Equations 1.1 – 1.3 are transformed from partial differential equations to algebraic equations by using finite difference approximations for the partial derivatives. The partial derivatives with respect to time for each segment are approximated by [11]

$$\frac{\partial Y}{\partial t} = \frac{Y_{i+1}^{n+1} + Y_i^{n+1} - Y_{i+1}^{n} - Y_i^{n}}{2\Delta t} \tag{1.18}$$

the spatial derivatives by

$$\frac{\partial Y}{\partial x} = \frac{Y_{i+1}^{n+1} - Y_i^{n+1}}{\Delta x} \tag{1.19}$$

and individual terms by

$$Y = \frac{Y_{i+1}^{n+1} + Y_i^{n+1}}{2} \tag{1.20}$$

Y represents pressure, mass flux and temperature. This discretization scheme is second order correct in time and first order correct in space. The pipeline is divided into N segments and N+1 nodes. 3N equations are derived for the pipe. The number of unknown values at time level n+1 is 3(N+1). Since one boundary value for each physical property is given at one end of the pipe, the number of unknowns reduces to 3N. The equations form a system of 3N nonlinear equations which can be solved by using the Newton-Raphson method.

4. RESULTS

Transmission of natural gas through a high pressure large diameter pipeline has been simulated using the numerical scheme presented in section 3.4. In sections 4.1 and 4.2 the system is characterized by a simple straight pipeline with a length of 140 km and internal diameter of 1 m holding a gas of molecular weight 18. Simulations were done for both rough and smooth pipes using different Reynolds numbers for the flow. In section 4.3 simulations were validated against operational data available from Gassco's network.

4.1 Rough pipe

For the rough pipe the roughness ε was set to 10 μm. Simulations were done under the following steady state conditions:

- Inlet pressure: 100 bar

- Inlet temperature: 30 °C
- Outlet mass flow: 500 kg/s
- Ambient ground temperature: 6 °C
- Overall heat transfer coefficient U: 2.84 W/(m²K)

Under the given conditions the Reynolds number of the flow is approximately $5*10^7$. With a surface roughness of 10 μm the dissipation correction factor is determined to be F=1.09. Simulations where run with and without correcting for turbulent flow in the dissipation term in the energy equation. The resulting temperature profiles along the pipe are shown in Figure 2. In the case where a correction factor (F=1.09) is applied to the dissipation term the temperature profile lies slightly above the profile for which no correction is applied (F=1). The difference between the two profiles increases with increasing pipe length. The difference between the two temperature profiles along the pipeline is shown in Figure 3 (bottom curve, U=2.84). The difference in outlet temperature is approximately 1 °C. A correction factor of 1.09 gives a higher value for the dissipation term in the energy equation, which in turn means a larger breakdown of mechanical energy to thermal energy and an increase in the gas temperature. If the pipe was thermally isolated so that the flow was adiabatic, then the effect of turbulent dissipation would be greater. This was investigated by setting the heat transfer coefficient U equal to zero. For the adiabatic case, the difference in outlet temperature when a dissipation correction factor (F=1.09) was applied was 1.6 °C. The difference in temperature profiles for adiabatic flow along the pipeline is also shown in Figure 3 (top curve, U=0).

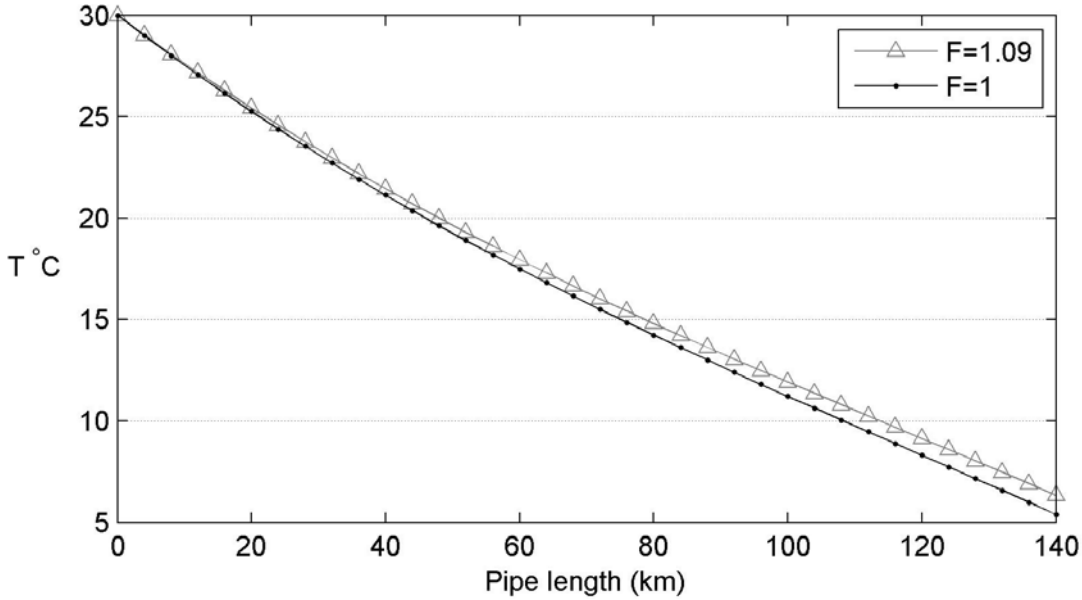

Figure 2: Temperature profile along the pipeline for a rough pipe. Temperature is slightly higher when the correction factor in the dissipation term is applied (F=1.09) compared to no correction (F=1). The difference in outlet temperature is approximately 1 ° C.

If the length of the pipe is increased, the temperature profile is similar to that in Figure 2. The temperature drops exponentially at the beginning of the pipeline before it eventually reaches an almost constant value for the remaining length of the pipe. For constant temperature the effect of the dissipation correction factor is merely a constant shift in the temperature profile. For the rough pipe under the given operating conditions, the effect of correcting for turbulent flow in the dissipation term is a difference in outlet temperature of 1 °C for a distance of 140 km or more.

Pressure is related to temperature through the real gas equation of state. The inlet pressure was given as a boundary value, while the outlet pressure was modeled. The difference in outlet pressure when

the correction is applied to the dissipation term was found to be less than 0.1 bar. The effect of turbulent dissipation on the pressure is therefore negligible.

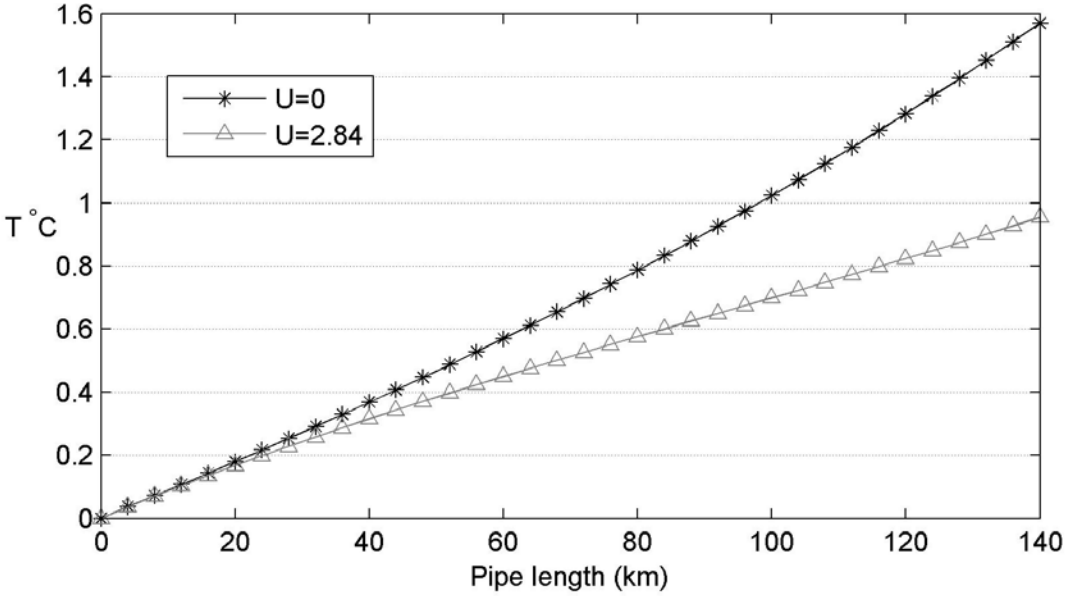

Figure 3: Difference in temperature along the pipeline when a correction factor in dissipation term is applied and no correction to the dissipation term is applied. The two different curves are for the adiabatic case (U=0) and for the heat conducting case (U=2.84). The resulting temperature difference in the heat conducting case is 1 °C and in the adiabatic case 1.6 °C.

4.2 Smooth pipe

For the smooth pipe the roughness ε was set to 1 μm. The operating conditions were slightly changed so that the dissipation correction factor F became less than unity for this case. Simulations were done under the following steady state conditions.

- Inlet pressure: 40 bar
- Inlet temperature: 30 °C
- Outlet mass flow: 200 kg/s
- Ambient ground temperature: 6 °C
- Overall heat transfer coefficient U: 2.84 W/(m²K)

The Reynolds number of the flow is just below $2*10^7$. The dissipation correction factor under these conditions is determined to be F=0.96. The resulting temperature profile along the pipeline for both the case where a correction (F=0.96) and no correction (F=1) is applied to the dissipation term is shown in Figure 4.

The effect of turbulent dissipation on the temperature for the smooth pipe under the given operating conditions is almost negligible. The resulting temperature difference at the outlet is 0.3 °C. When a correction factor of 0.96 is applied to the dissipation term the temperature of the gas becomes slightly lower than when no correction is applied. If the flow was considered to be adiabatic then the difference in outlet temperature was found to be approximately 1 °C. For a smooth pipe the dissipation correction factor is for typical operating conditions always less than and close to unity. The effect of turbulent dissipation is much less on smooth pipes compared to rough pipes.

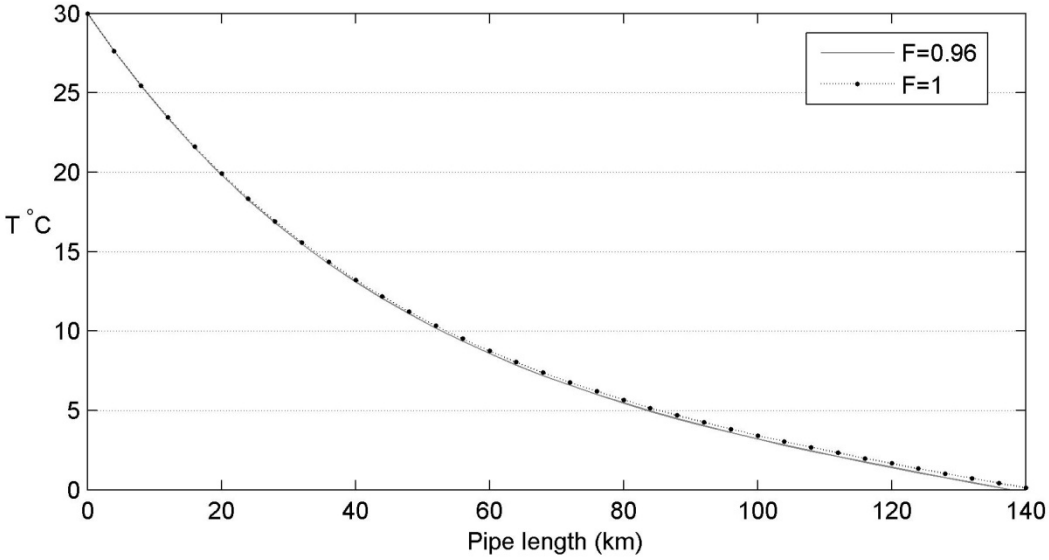

Figure 4: Temperature profile along the pipeline for a smooth pipe. Temperature profile is almost identical when the correction factor in the dissipation term is applied. The difference in outlet temperature is 0.3 °C. The effect of considering turbulent dissipation in this case is almost negligible.

4.3 Validation against operational data

The implemented model is validated against real operational data from Gassco's network. The model is tested on a 50 km onshore pipeline. The pipeline has an internal diameter of 1 meter. Operational data are collected from measurement sensors which are located at the inlet and outlet of the pipe. At the inlet the model is given pressure and temperature as boundary values, while at the outlet the mass flow is given. Modeled inlet mass flow and outlet pressure and temperature are compared to measured values. Results for inlet mass flow and outlet pressure are shown in Figure 5 and 6 respectively. The resulting outlet temperature profile is shown in Figure 7. The modeled temperature lies slightly above the measured value. The dissipation correction factor F for the conditions in this flow is 0.96. By considering turbulent flow and applying the correction to the dissipation term in the energy equation the result for the temperature lies closer to the measured value. The difference in modeled temperature when applying the dissipation correction factor is approximately 0.25 °C. Although there is a certain degree of uncertainty in the modeled temperature, the results demonstrate that considering turbulent flow when solving the energy equation gives slightly different results for the temperature in the flow. The surface of the pipeline is coated, meaning the roughness is approximately 3 μm. This implies a smooth pipe which means the effect of the dissipation correction factor is not so large. For a rougher pipe the difference in temperature would be more noticeable.

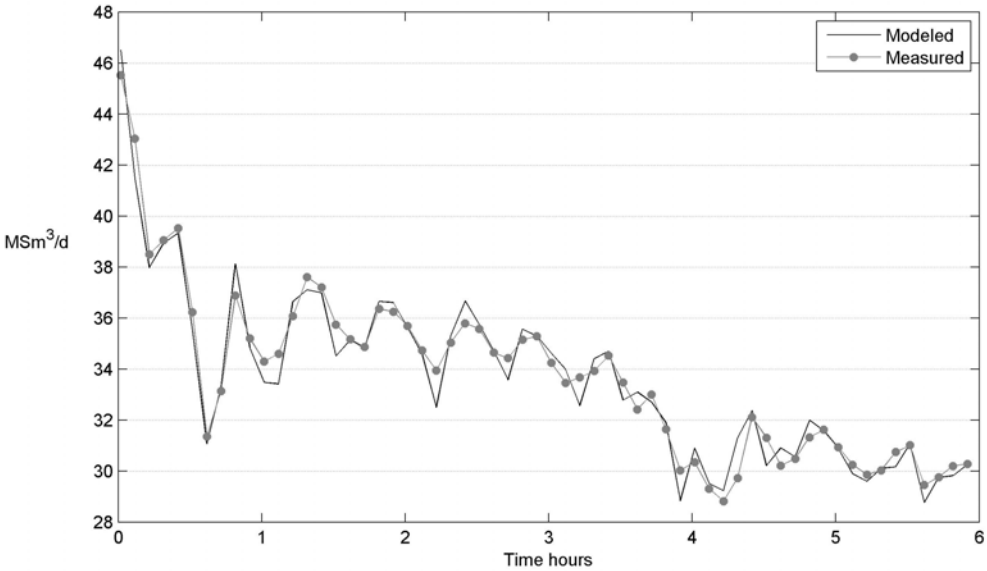

Figure 5: Inlet mass flow in the 50 km pipeline. Modeled values are compared to measured operational data. Mass flow given as million standard cubic meters per day.

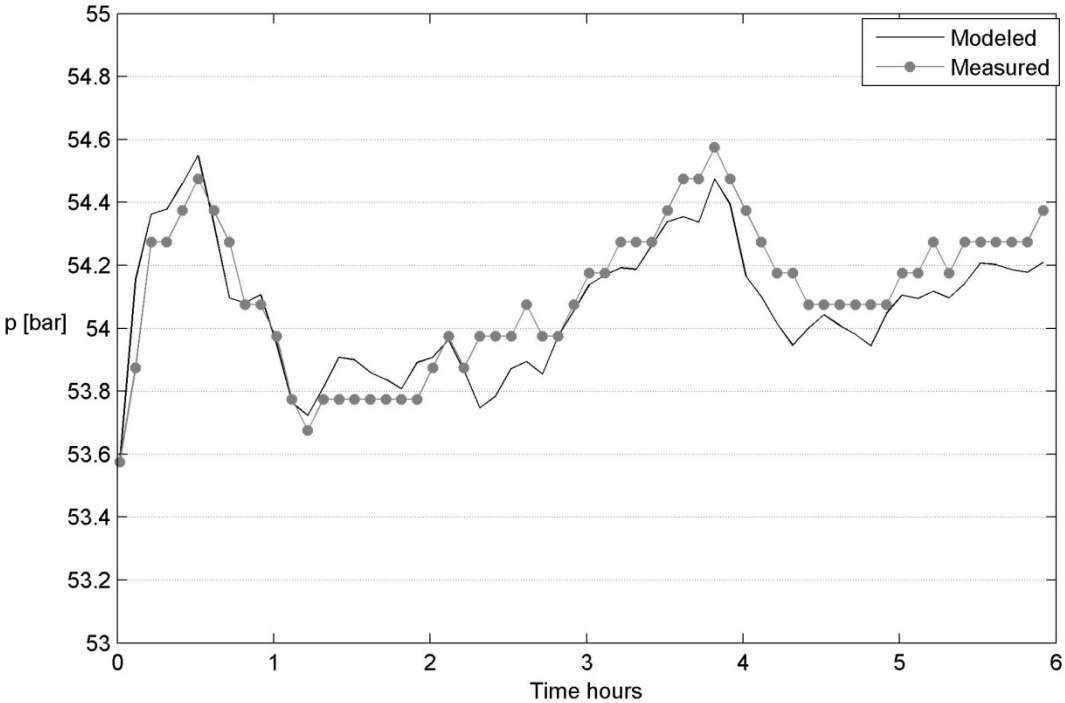

Figure 6: Outlet pressure in the 50 km pipeline. Modeled values are compared to measured operational data. Pressure given in bars.

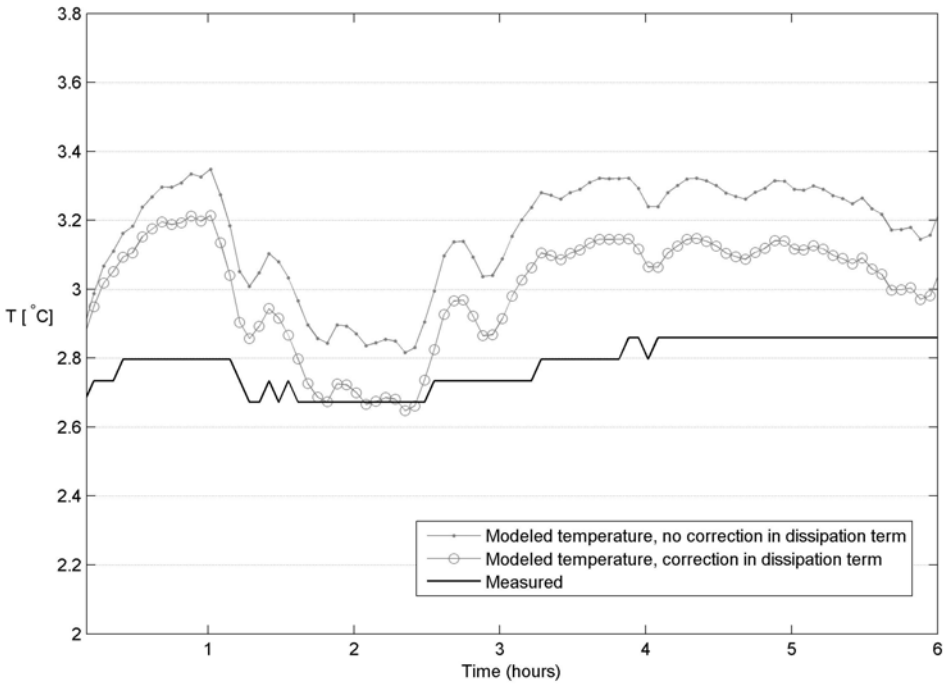

Figure 7: Results for outlet temperature in the 50 km pipeline. By considering turbulent flow and including the correction factor in the dissipation term in the energy equation the modeled temperature is closer to the measured temperature.

5. CONCLUSION

In this paper we have considered the effect of turbulent dissipation on one-dimensional flow in gas pipelines. The energy equation commonly used in one-dimensional gas transport is valid for laminar flow. The transportation of natural gas in high pressure pipelines is always turbulent. Usually, the only modification made to the energy equation which is valid for laminar flow is to replace the laminar friction factor by the turbulent friction factor f. But, as pointed out by Ytrehus [8], a turbulent velocity profile should be considered when transforming the energy equation from a three-dimensional version to a one-dimensional version. By doing so it is shown that the dissipation term should by multiplied by a non dimensional factor which depends on the Reynolds number of the flow and the surface roughness of the pipe.

For typical operating conditions in natural gas pipelines the dissipation factor F(Re,ε) is either slightly larger or smaller than unity, depending on the characteristics of the flow and the pipe. If the pipe is rough (ε equal to 10 μm or larger), then turbulent dissipation will have a noticeable effect on the temperature in the pipe. Depending on characteristics of the pipe the effect of not applying a correction could be an error in temperature close to 1 °C. If the flow can be considered adiabatic then the error will be even larger. The effect of turbulent dissipation is most noticeable for high Reynolds number flows. For smoother pipes (ε in range 1-5 μm) the dissipation factor is so close to 1 that it has negligible effect on the temperature. The effect of turbulent dissipation was investigated on one of Gassco's high pressure transmission pipelines. The result was a slightly more accurate value for the modeled outlet temperature.

For transportation of natural gas in large scale pipelines which have been coated, the effect of turbulent dissipation is in most cases negligible and one can safely use the laminar expression for the

dissipation term. But one should however be aware of the effect and that under certain conditions, such as for large Reynolds numbers and rough pipes, is should be included if one is interested in getting as accurate results as possible. The effect is easily implemented into any code used to solve one-dimensional gas flow and does not give any noticeable increase in CPU time when solving the governing equations.

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Nomenclature

A – cross section area of the pipe, m^2

C_f - skin friction

D – diameter of pipe, m

F – turbulent dissipation factor

f – friction factor

g – gravitational acceleration, m/s^2

h – enthalpy, J/kg

k – thermal conductivity, $W/(m*K)$

p – gas pressure, Pa

R – specific gas constant, J/kg

R^+ - dimensionless pipe radius

Re – Reynolds number

T – temperature, K

T_a - ambient temperature, K

t – time, s

U – total heat transfer coefficient, $W/(m^2K)$

u – flow velocity, m/s

W – frictional force per unit length of pipe, N/m

Z – gas compressibility factor

x – spatial coordinate, m

y_0^+ - dimensionless sub layer thickness

ε – pipe surface roughness, m

θ – angle of inclination of the pipe, radian

κ – von Karman constant

ρ – density of the gas, kg/m^3

τ - shear stress, N/m^2

Φ – dissipation, $N/(m^2s)$

Ω – heat flow into pipe per unit length and time, $J/(m*s)$