

# WGCPARIS2015

WORLD GAS CONFERENCE  
*"GROWING TOGETHER TOWARDS A FRIENDLY PLANET"*



26th World Gas Conference | 1-5 June 2015 | Paris, France

## **OPTIMISATION OF LONG DISTANCE GAS PIPELINE PROJECTS**

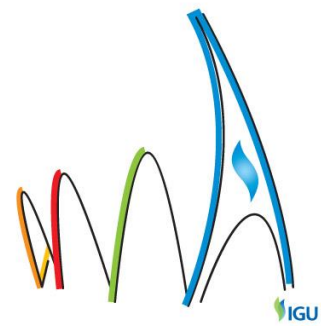
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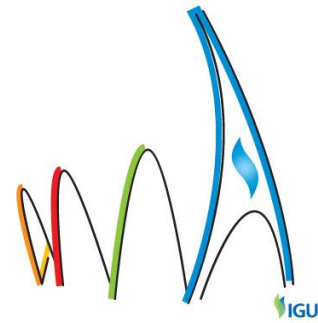
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### 1. Introduction

Every project for long-distance natural gas transportation through pipelines must comply with basis parameters, such as flow, input pressure at the pipeline start point, minimum discharge pressure at the pipeline delivery point, route along which the pipeline will be laid (flatlands, mountain ranges, subsea lines, etc.), and any other conditions set forth in the design basis.

With these foundations and with the relevant mathematical formulas and calculation software, the suitability of the project for the transportation of the required natural gas flow at the pressure foreseen in the design basis is ensured. However, this is not enough, since this calculation method gives no indication of transportation costs.

The key objective of this type of project is to transport gas at a "minimum cost", and the methodology frequently used does not result in a minimum cost project, which we shall call "optimum project".

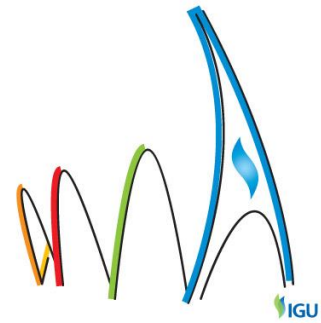
The fact that we are looking for a minimum cost makes us evaluate the cost of all the components which comprise the gas pipeline, maintenance costs, and the cost relative to the energy required to operate the compressor stations, as well as all other costs inherent to the system during its previously defined life cycle.

Natural gas being a compressible fluid is clearly more suitable for pipeline transportation at a very high pressure, thus resulting in a smaller pipeline diameter, which, in turn, means lower costs. However, transportation at a very high pressure also means that higher energy consumption is needed to achieve the required compression and also compressor equipment of greater size is required, all resulting in higher costs. Another variable involved is the number of compressor stations, due to the fact that for a larger number of compressor stations there will be lower pressure increase for the same given transport distance. This, on the one hand, increases costs since there are more compressor stations, and on the other hand, the cost is reduced on account of the pressure drop being diminished.

When trying to solve the problem mathematically, we find that the number of equations describing the problem is lower than the number of unknown values to be defined, thus the equation system allows for an infinite number of solutions. Amongst those infinite solutions, and with the assistance of cost functions derived from the cost of the components of the installation, the optimum project is obtained, by means of the mathematical calculation, developed by Lagrange, the mathematician.

Finally, a set of relatively easy to handle formulas is reached, applicable to many practical cases, which allows the definition of variables with their optimum value, and its consequent minimum costs. After adequate analysis is performed, is a function of the pipeline diameter, thus:

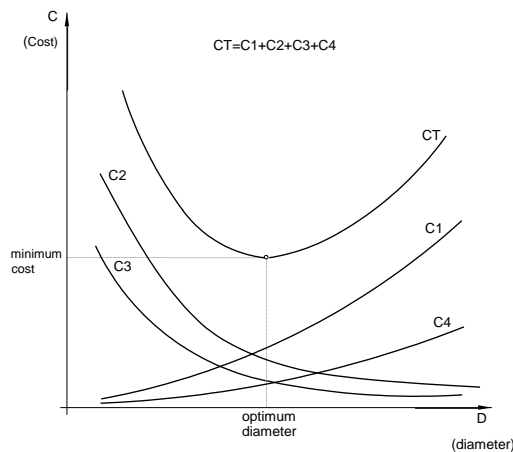
- Cost of line pipe, valves and fittings:  $C_1 = f_1 (D)$
- Cost of energy (present value) required for compressor stations:  $C_2 = f_2 (D)$
- Cost of compressor stations:  $C_3 = f_3 (D)$



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- Cost of gas (present value) needed for filling gas pipeline:  $C_4 = f_4(D)$
- Total cost:  $CT = f_t(D) = C_1 + C_2 + C_3 + C_4$ ,

The representation of these functions is shown as follows



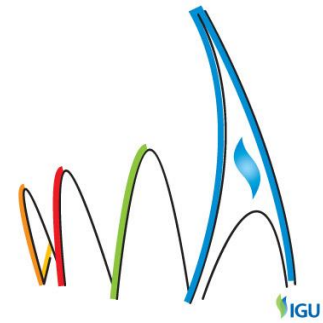
As can be seen in the graph, optimum diameter D is obtained, corresponding to the minimum cost.

#### Indeterminate Compressible and Isothermal Flow

Any long-distance transportation pipeline in general consists of the line pipe, a head end compressor station and successive intermediate compressor stations. In each section, the pipe will have a maximum operating pressure  $P_1$  at the beginning (compressor discharge) and a minimum operating pressure  $P_2$  at the end of the pipe (suction) due to pressure loss caused by the flow.

It should be noted that the complete development of the mathematical formula and constants employs the International System of Units (SI Units).





For the purposes of this optimisation calculation, it is assumed that isothermal steady-state gas flow<sup>1</sup> is established all along the pipe interior. The simplified equation for long pipes expressing the mass flow of transported gas is as follows:

$$C_p^2 = \frac{\pi^2 D_i^5 \chi}{16 RTfL} (P_1^2 - P_2^2) = \frac{\pi^2 D_i^5 P_1^2 \chi}{16 RTfL} \left[ 1 - \left( \frac{P_2}{P_1} \right)^2 \right]$$

Where:

- $C_p$ : mass flow [kg/s]
- $D_i$  : internal diameter [m]
- X: number of compressor stations including the head end
- R : particular constant of gas [J/kg.°K] (corrected with compressibility factor Z)
- T : equivalent gas temperature to consider it isothermal [°K]
- f : friction factor obtained with Moody or Churchill formula
- L : total length of pipe [m]
- $P_1$ : maximum pressure in pipe interior at inlet [Pa]
- $P_2$ : minimum pressure in pipe interior (at delivery point) [Pa]

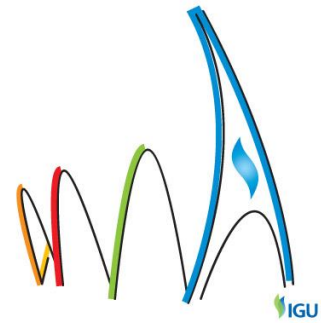
There follows an equation with four unknown quantities, which allows for infinite solutions. The question is to select among the infinite solutions the one that will keep the cost of the entire facility at a minimum, i.e. the "optimum solution". To this end, cost functions of all items intervening in the installation must be created. As was stated above, installation costs are divided into four large groups: cost of pipe  $C_1$ , cost of energy used in compressor stations  $C_2$ , cost of compressor stations  $C_3$ , and cost of initial filling gas of pipeline  $C_4$ .

#### Determining the equivalent temperature of transported gas

The equivalent temperature of transported gas (to be applied in the isothermal formula) can be determined by the equation<sup>2</sup>:

<sup>1</sup> See book "Flow of Fluids through Valves, Fittings and Pipes" –Crane Co Engineering Division, USA.

<sup>2</sup> Source: 8<sup>th</sup> International Gas Conference (Stockholm 1961) "The calculations of pipelines and prerequisites for choosing optimum gas transmission conditions" Authors: A.V.Alexandrov, B.V. Barabash, I.E.Khodanovich – International Gas Union-IGU.



$$t_{\text{prom}} = t_t + \frac{t_1 - t_t}{\frac{U \cdot \pi \cdot D \cdot L}{C_v \cdot C_p}} \left( 1 - e^{-\frac{U \cdot \pi \cdot D \cdot L}{C_v \cdot C_p}} \right)$$

where:

- $t_1$ : gas temperature at the beginning of the section [°C]
- $t_t$ : soil temperature at the depth of the pipeline [°C]
- $U$ : heat transfer factor from gas to soil [W/m<sup>2</sup>. °C]
- $C_p$ : gas heat capacity [J/kg. °C]
- $D$ : outer diameter [mm]

## 2. Cost Functions

In order to simplify the mathematical procedure to obtain functions, the total installation cost is divided into four large groups of variables.

### 2.1. Cost of pipeline "C<sub>1</sub>"

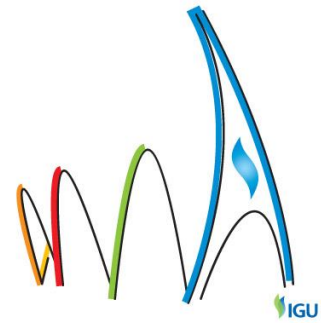
The cost of the pipeline depends on the cost of materials (i.e. the steel pipe, and the corrosion protective insulating, line block valves, various pipe fittings, etc.) and cost of installation.

The latter comprises such wide ranging topics as right of way, engineering, surveying, terrain clearing and grading, stringing, joint welding or other method, supports, ditching in case of buried pipe, final tests, and all costs related to manpower and equipment required for installation activities. By means of a market research study the various costs are obtained as a function of the diameter, which for practical purposes is assumed to be linear functions, valid within a given interval of diameters

To obtain this function,  $C_1 = f(D)$ , and for the purposes of a simple formulation, it is useful to divide intervening costs in two large groups, one proportional to the weight of the pipe and another group proportional to the diameter of the pipe. Thus:

$$C_1 = C_{11} + C_{12}$$

$C_{11}$ : Includes costs which are dependent on the pipeline weight, such as pipe material, welding and, to some extent, manpower and installation.



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$C_{12}$ : Includes costs which are dependent on the surface of the pipe such as corrosion protective insulating or thermal coating, manpower and right of way and ditching and backfilling, as well as land restoration, etc.

Also, in both cases maintenance costs of the duct throughout its life cycle must be borne in mind.

$$C_{11} = \kappa_{11} \cdot \pi \cdot D_m \cdot e \cdot L \cdot \gamma_F$$

Where:

- $\kappa_{11}$  = market constant for steel cost [\$/N]
- $\gamma_F$  = specific weight of pipe material [N/m<sup>3</sup>];
- $D_m$  = average pipe diameter  $\left[ \frac{D_{ext} + D_{int}}{2} \right]$  [m]
- $e$  = pipe wall thickness [m]
- $L$  = total length of pipeline [m]

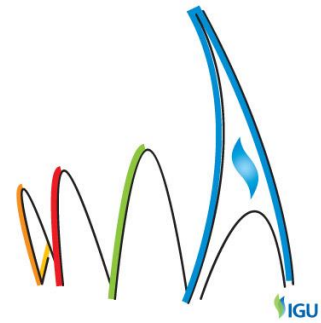
Furthermore, bearing in mind the expression of the strength of thin cylindrical casings materials (a formula usually adopted by the standards in force)

$$\sigma_{ad} = \frac{P_1 D}{2e} \times F$$

Where  $\sigma_{ad}$  is defined as: allowable stress of material; which is generally the steel yield stress of the pipe affected by a safety factor called factor "F" or "design factor", with values ranging between 0.8 and 0.4 dependent on the location of the pipeline and the population density of the adjacent area, i.e. the so-called "class location unit"<sup>3</sup>

#### Factor "K<sub>D</sub>"

<sup>3</sup> The class location unit is an area extending 200 meters/quarter mile on either side of the centerline of any continuous section of the pipeline of 1600 meters/one mile (see NAG 100 standards, Section 5) or also ANSI/ASME B31.8.



In the expressions of cost and flow, the average diameter  $D_m$  (in the simplified weight formula) and the outer diameter  $D$  (in the Barlow formula) and the inner diameter  $D_i$  (in the flow formula) are shown.

One of these three variables  $D_m$ ,  $D_i$  and  $D$  has to be removed, which is difficult. To do so, a new "variable" of very little variation is created, that is, a "quasi-constant" called  $K_D = D_i/D$ .

In fact, to avoid developing formulas in three different diameters, it is defined  $K_D = D_i/D$ . that is, the ratio of the inner diameter to the outer diameter of the pipe.

This factor ( $K_D$ ) enables to keep only the outer diameter  $D$  in the expression (initially estimating the  $K_D$  and then checking it), and thus "drag" only one unknown value.

$$\text{Where: } D_m = \frac{D_i + D}{2} = \frac{D \left( \frac{D_i}{D} + 1 \right)}{2} = \frac{D(K_D + 1)}{2}$$

### Cost of installed pipe

Replacing  $D_m$  and  $e$ , the expression  $C_{11}$  (the portion of the value of the installed pipe which cost is proportional to the steel weight) results in:

$$C_{11} = K_{11} \frac{1 + K_D}{2} \frac{\pi D^2 P_1}{2 \sigma_{ad}} \gamma_F L$$

$C_{12}$  is a function of the external surface of the pipe and for calculation purposes is assumed as a linear function with the external surface of pipe, valid for a given interval of diameters.

In this cost, the corresponding corrosion protective coating is included with a coefficient of  $K_{12}$ , and the consequent manpower involved in laying of pipe, ditching, repairs and other aspects relative to installation, with a coefficient of  $K_{13}$ .

$$C_{12} = (K_{12} + K_{13}) \pi D L$$

Where:





- $K_{12}$ : Market constant corresponding to the cost of thermal and/or anti-corrosive insulation coating per area unit [ $\$/m^2$ ].
- $K_{13}$ : Market constant corresponding to the cost of pipe installation per area unit [ $\$/m^2$ ].

Then the  $C_1$  function results in:

$$C_1 = K_{11} \frac{1 + K_D}{2} \frac{\pi D^2 P_1}{2\sigma_{ad}} \gamma_F L + (K_{12} + K_{13}) \pi DL$$

### Extra thickness required for Corrosion

In those cases where gas can cause serious corrosion ("acid" gases basically as a result of sulphur compounds -  $H_2S$ , mercaptans and organic sulphides- or  $CO_2$ , generally corrosive in presence of water), it is advisable to define additional thickness to that resulting from pressure calculation (eg, adding 2-3 mm). Clearly, this extra thickness shall not be included in the strength calculation (stress), i.e.  $C_{11}$ , but certainly it shall be considered when calculating the cost of installation  $C_{12}$ .

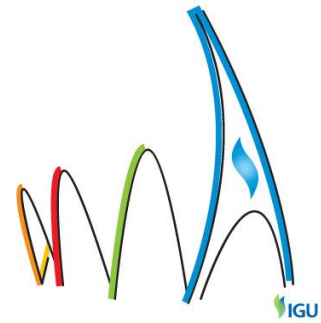
The additional weight under extra thickness  $e_c$  results in:

In this case the Cost  $C_1$  will be modified as follows:

$$C_1 = K_{11} \frac{1 + K_D}{2} D^2 \left( \frac{P_1}{2\sigma_{ad}} \right) \pi \gamma_f L + (K_{12} + K_{13} + K_{14}) \pi DL$$

## 2.2. Cost of power consumption in compressors " $C_2$ "

This cost may be considered as a function of the power required in compressor plants and the operating time foreseen for the facility. The figure below shows an approximate curve of the variation in energy cost consumption as a function of the gas pipeline diameter  $D$ . In order to add it to the total costs equation, the rates of power consumption during life cycle



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should be updated with a convenient rate (present value), since payments are made regularly throughout the life of the pipeline.

The expression employed for calculating Power needed for compressing gas, allowing for an isentropic compression, multiplied by a factor reflecting the of said compression, is as follows:

$$N = C_{\rho} \frac{K}{K-1} \frac{R'T}{\eta_c} \left[ \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} - 1 \right] \chi \cdot \left( \frac{Zc_m}{Zc_2} \right)$$

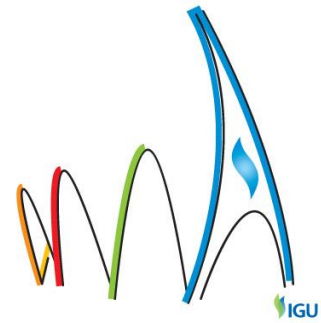
Where:

- N = required power (watts)
- R = Particular constant of gas (includes Z compressibility factor)  $\left[ \frac{J}{Kg^{\circ}K} \right]$
- T = Gas temperature at suction [ $^{\circ}K$ ]
- $P_1$  = Absolute pressure at the discharge of the compressor plant [Pa]
- $P_2$  = Absolute pressure at the suction of the compressor plant [Pa]
- X = number of compressor stations along the pipeline (including head end)
- Zc = medium compressibility factor  $Zc_m = (Z_1 + Z_2) / 2$  and  $Zc_2$  suction)

The compressors are operated by engines -which may be either gas alternative motors, such as gas turbines or electric motors-. These engines have also their own performance.

$$C_2 = K_{o2} K_2 \frac{N}{\eta_m} t$$

Where:



- $K_2$ : Constant for the cost of energy.
- $t$ : Estimated operation time of the compressor facilities.
- $K_{02}$ : Cost updating coefficient.

On the basis that energy is paid for "n" periods of time (generally on a monthly basis), throughout the total life of the facility, the cost of energy is multiplied by a coefficient that allows estimation of the present value of the cost  $C_2$ . Assuming that energy is paid per month, at a due date, the monthly interest rates is  $i$ , and the period number (months) is  $n$ , the value of  $K_{02}$  is:

$$K_{02} = \frac{(1+i)^n - 1}{n[i(1+i)^n]}$$

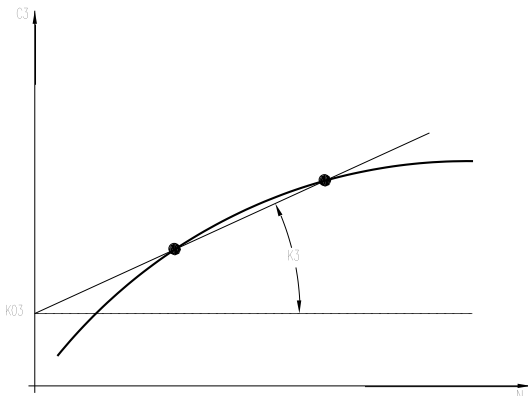
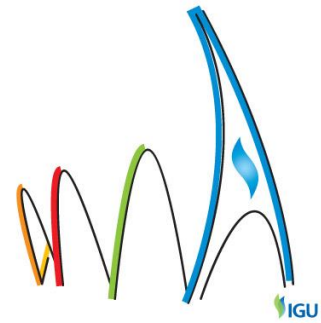
When replacing  $C_2$  in the expression of the cost of compression power (considering the specific gas constant  $R$  is affected by definition of suction compressibility coefficient  $Zc_2$ ),  $C_2$  is:

$$C_2 = \frac{K_{02}K_2t}{\eta_m\eta_c} \frac{K}{K-1} C_p RT \left[ \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} - 1 \right] X.Z_{Cm}$$

### 2.3. Cost of compressor stations "C<sub>3</sub>"

This cost may be considered as a non-linear function of the compression power including the power required to operate the pipeline and the backup power (installed power). Also, the cost of ancillary facilities must be included (separation filters, valves, air-coolers, pumping systems, flares, etc.), maintenance and / or parts replacement of plant throughout its life. To simplify calculations, a valid linear function is assumed within a certain range of powers.

The approximate curve of said function, is shown below:



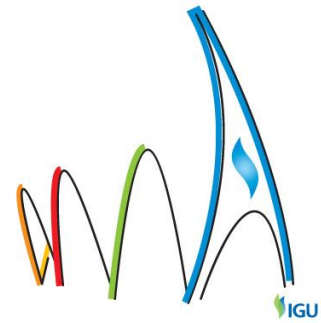
$$C_3 = (K_{03} + K_3 N)\chi$$

Where:

- $N$  = installed power in each compressor [W]
- $K_{03}$  = Figure which is, to some extent, independent of compression power, and mainly represents "structure" costs related to ancillary facilities and land surface. All these costs are indirectly related to installed power [\$].
- $K_3$  = Market constant for the average power unit cost of compression stations (motor units + installed and operating compressors) for the resulting power range [\$/W].

$$C_3 = K_{03}\chi + K_3 \frac{K}{K-1} \frac{C_p RT}{\eta_c} \left[ \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} - 1 \right] \chi \cdot Zc_m$$

As the power to be applied on each plant is not known at the beginning, approximate values of  $K_{03}$  and  $K_3$  are used to test such power. The cost verification (per plant) may be made with the following theoretical and practical expression (approximate) obtained as a simple regression of market data, once the power involved in each plant is known:



$$C_3 = C_0 \sqrt{N}$$

Where N is the total power installed on each compressor plant, expressed in Watts and  $C_0$  a market constant [\$/ W<sup>1/2</sup>]

#### 2.4. Cost of gas required to fill pipeline "C<sub>4</sub>"

This cost is a function of the gas rates (and their quality upon being injected into the pipeline) required for initial filling that will enable pipe operation, necessary as "working capital", which is added at the start of the project and which in theory would be compensated at the end of the estimated life cycle of the project. For this reason the present value of the immobilised capital must be considered.

Clearly, this directly depends on the amount of gas added in, that is to say, the inner pipe size (i.e. its diameter) and the transport pressure.

As stated above, the cost of the gas volume needed to fill in the pipeline is a function of the interior size (volume) of the pipe (i.e. its length and inner diameter) and also the transport pressure.

Considering the compression ratios commonly used in gas compression of main pipelines (in general these ratios range between 1.3 and 2), such simplified cost.

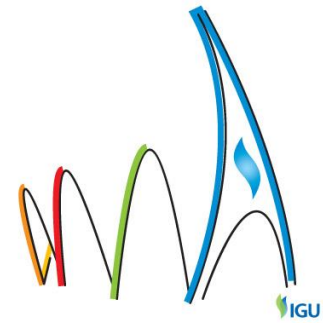
The approximate cost of initial filling gas (also called initial "line pack") will be:

$$C_4 = K_{04} K_4 K_D^2 \frac{2P_1}{9RT} \pi D^2 L$$

Where:

$K_4$  = Cost coefficient of gas in  $\left[ \frac{\$}{Kg} \right]$  which reflects the cost of cubic meter of gas for the quality set at the gas pipeline inlet (i.e. treated and conditioned).

$K_{04}$  = Coefficient of cost update of immobilised capital during life cycle of gas pipeline, and recuperated at the time of closing down, which value may be obtained with the following formula:



$$K_{04} = 1 - \frac{1}{(1+i)^n}$$

### 2.5. Total cost of project "C<sub>T</sub>"

Having obtained the four cost functions, they are added up to reach the total cost (present value) of the project

$$C_T = C_1 + C_2 + C_3 + C_4 = f(D)$$

The minimum C<sub>T</sub> is obtained as a derivative of diameter  $\frac{\partial C_T}{\partial D} = 0$  and from this the optimum diameter is found. The graphic representation of this is to be found in graph of the Introduction.

Theoretical simplification shown to obtain the optimum Diameter, is not possible through a direct approach in the proposed calculation scheme because cost functions C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub> and C<sub>4</sub> do not result from the function of a single variable to be optimised "D", but are functions of several variables.

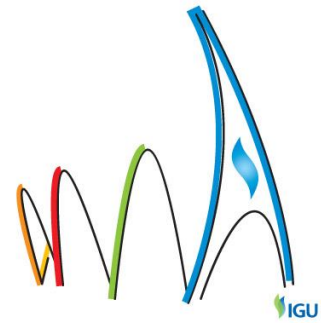
In this case, the actual possibility of finding an optimum is directly related to the method employed. In order to optimise several variables subject to technical conditionings, the Lagrange Multiplier rule or "conditioned maximum and minimum values" is employed, the mathematical rigor of which requires no further justification. The mathematical formulation of the optimisation problem raised is explained below.

The full expression of C<sub>T</sub> for a pipeline system is the following:

$$C_T = \left[ K_{11}(1 + K_D) \frac{\pi \gamma_f}{4 \sigma_{ad}} + \frac{2}{9} K_{04} K_4 K_D^2 \frac{\pi}{RT} \right] P_1 D^2 L + (K_{12} + K_{13}) \pi D L + \dots$$

$$\dots + \frac{K}{K-1} \left( \frac{K_{02} K_2}{\eta_m \eta_c} + \frac{K_3}{\eta_c} \right) C_p RT \left[ \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} - 1 \right] X Z_m + K_{03} X$$

To achieve optimisation, the objective is searching the "minimum" of C<sub>T</sub> function (i.e. the minimum cost of installing and operating the pipeline system) subject to the function of pressure loss for isothermal compressible flow at steady state.



### 3. Variable Optimisation

Once the cost function has been obtained  $C_t = f\left(D; P_1; \frac{P_1}{P_2}; \chi\right)$ , the aim is to optimise the variable parameters Diameter:  $D$ , Maximum operation pressure:  $P_1$ , Compression Ratio:  $P_1/P_2$  for the compressor stations, and number of compressor stations on the line:  $\chi$ ; all of them conditioned by the isothermal flow equation for steady regime:

$$C_p^2 = \frac{\pi^2}{16} \frac{K_D^5 D^5 P_1^2 \chi}{RTfL} \left[ 1 - \left( \frac{P_1}{P_2} \right)^{-2} \right]$$

Using the Lagrange Multiplier Method the optimum values are obtained, as shown below:

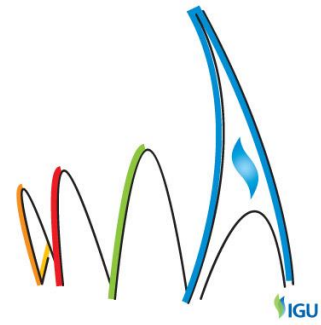
$$C_T = f\left(D, P_1, \frac{P_1}{P_2}, \chi\right) \quad (\text{cost function})$$

$$\varphi = C_p^2 - \frac{\pi^2}{16} \frac{K_D^5 D^5 P_1^2 \chi}{RTfL} \left[ 1 - \left( \frac{P_1}{P_2} \right)^{-2} \right] = 0 \quad (\text{border condition equation})$$

Lagrange system (with  $\lambda$  Lagrange multiplier)

$$1) \frac{\partial C_T}{\partial D} + \lambda \frac{\partial \varphi}{\partial D} = 0$$

$$2) \frac{\partial C_T}{\partial P_1} + \lambda \frac{\partial \varphi}{\partial P_1} = 0$$



$$3) \frac{\partial C_T}{\partial \left( \frac{P_1}{P_2} \right)} + \lambda \frac{\partial \phi}{\partial \left( \frac{P_1}{P_2} \right)} = 0$$

$$4) \frac{\partial C_T}{\partial \chi} + \lambda \frac{\partial \phi}{\partial \chi} = 0$$

In the case of gas transmission systems, the unknown expressions to obtain are:

- $D$  : Outer pipe diameter
- $P_1$  : Maximum operating pressure of the pipe
- $P_1/P_2$  : Compression ratio
- $\chi$  : Number of compressor plants, including the head end.

We have obtained four equations applying LaGrange's method, which together with the border condition equation conforms a five equation systems with five unknown values.

By solving the equation system (removing  $\lambda$ , the Lagrange variable), the following expressions are obtained:

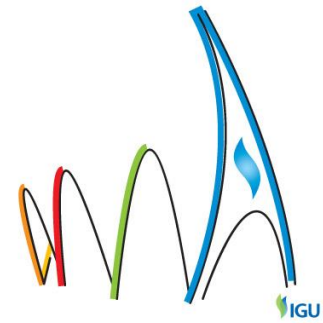
$$(I) \left( \frac{K_{02}K_2t}{\eta_m\eta_c} + \frac{K_3}{\eta_c} \right) C_\rho RT \left\{ \frac{1}{2} \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} \left[ \left( \frac{P_1}{P_2} \right)^2 - \frac{3K-1}{K-1} \right] + \frac{K}{K-1} \right\} - K_{03} = 0$$

$$(II) D = \left\langle \left\{ \frac{K}{K-1} \left( \frac{K_{02}K_2t}{\eta_m\eta_c} + \frac{K_3}{\eta_c} \right) C_\rho RT \left[ \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} - 1 \right] + K_{03} \right\} \left( \frac{K_{11} \frac{1+K_D}{8\sigma} \gamma_f + \frac{K_{04}K_4K_D^2}{9RT} \right)^2 \frac{16C_\rho^2 RTf}{K_D^5 \left[ 1 - \left( \frac{P_1}{P_2} \right)^{-2} \right]} \right\rangle^{\frac{1}{4}}$$

$$(III) P_1 = \frac{K_{12} + K_{13}}{\left( K_{11} \frac{1+K_D}{8} \frac{\gamma_f}{\sigma_{ad}} + \frac{K_{04}K_4K_D^2}{9RT} \right) D}$$

$$(IV) \chi = \frac{16C_\rho^2}{\pi^2 K_0^5} \frac{RTfL}{D^5 P_1^2 \left[ 1 - \left( \frac{P_1}{P_2} \right)^{-2} \right]}$$





To begin solving the case, from equation (I) the compression ratio  $\frac{P_1}{P_2}$  is obtained; then with this pressure equation ratio (II) the outer diameter  $D$  is obtained. Afterwards, from equation (III) the pipe maximum pressure  $P_1$  is obtained and from equation (IV), the number of compressor stations (including head end) is obtained.

During the above-explained solving cycle, the factor ( $K_D$ ), the friction factor  $f$ , the cost constants  $K_3$  and  $K_{O3}$  which depend on the resulting power, etc. must be checked, all of them adopted at the beginning of the calculation process.

It should be noted that the outer diameter was chosen for the purposes of optimisation (to avoid "dragging" more variables). The outer diameter  $D$  and pressure  $P$  make feasible to calculate theoretical thickness and inner diameter. In this same manner, the inner diameter may be inferred with the coefficient  $K_D$  and the outer diameter.

Proceeding thus, it is possible to determine the four sought variables which bring the total cost of the project to a minimum value and are therefore referred to as theoretical "optimum".

Knowing now the calculated optimum variables, the pipe wall thickness can be then calculated with Barlow's formula (ANSI B31.8):

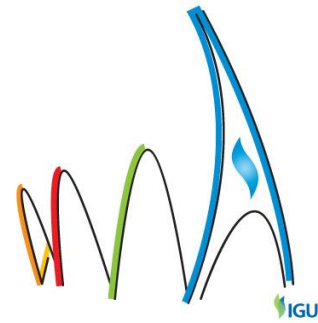
$$e = \frac{P_1 D}{2\sigma_{ad}}$$

The allowable stress  $\sigma_{ad}$  is obtained from the yield stress  $\sigma_f$  of the material affected by safety factor  $K_5$  (ranging between 0.8 and 0.4) usually established in the specifications by the product of the Design Factor  $F$  which takes into account safety standards (class location units based on population density), type of pipe construction (Factory-Made Welding Efficiency Factor  $E$ ), operating conditions (Temperature Factor  $T$ ), among others:

$$\sigma_{ad} = F.E.T.\sigma_f$$

The results arise from a mathematical optimisation; the approach to commercial or regulatory values poses a new optimisation in terms of variables of common or standardized use (eg, "commercial" outer diameters of pipes), discrete variables (entire number of compressor plants), or depending on present regulations (restrictions on maximum operating pressures), usually achieved by a series of detailed calculations of the pipeline being projected.

For pipe diameter  $D$ , the closest calculated optimum diameter, either larger or smaller, should be selected within the standardized series.



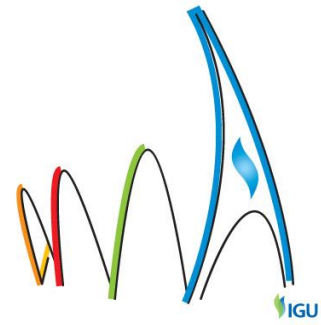
If the theoretical Optimum Diameter is larger than the possible ordinary manufacturing pipes, the project is divided into two parallel pipes with half the flow each and the optimum variables are recalculated.

#### Methodology of "Practical" Adjustment of the OPTIMISATION Variables

Considering that from the theoretical formulation, the variables  $D$  and  $P_1$  as well as theoretical thickness are obtained, commercial or manufacture standard dimensions must be selected. It may be called practical or trade adjustment of the results. Same operations must be conducted on the result of  $X$ , i.e. the number of plants, which must be a discrete number.

Therefore, for the practical resolution of a pipeline design under the method of optimisation the following logical sequence must be followed:

- 1) Solve successfully the complete equations to obtain  $P_1$ ;  $X$ ;  $D$  and  $P_1/P_2$  (the adopted  $K_D$  ratio, the friction factor and cost constants  $K_{03}$  and  $K_3$  of the compression plant must be proved).
- 2) Based on the theoretical optimum Diameter  $D$ , select the largest and the smallest commercial diameter.
- 3) Based on the "non-integer" number of  $X$  compressor stations, select the largest and smallest integer number.
- 4) Define different projects by selecting the one of largest diameter and fewest number of compressor plants and vice versa, and one of largest diameter and largest number of plants and vice versa.
- 5) Calculate pressure losses of a  $L/X$  section for the above-mentioned commercial alternatives, applying isothermal steady flow equation (in each  $D$  and  $X$  case). In this way, the compression ratio  $P_1/P_2$  will be found for each case, using the  $P_1$  obtained from resistant calculation obtained for the selected thickness.
- 6) For all cases (with each commercial diameter,  $P_1$ ,  $P_2$  and  $X$  integer) with the expressions of initial cost, calculate the total cost of each alternative and select as the optimum alternative that one with the lowest cost or, under similar solutions in terms of cost, select the one which best meets any additional requirement for the requested purposes.
- 7) Finally solve the selected optimum alternative by "small sections" (segments) and adjust the location of compressor stations in order to have similar power in all compressor plants (i.e. achieving similar compression ratios in all compressor plants).



#### Final adjustment of the variables for the projected system

It should be mentioned that the "optimum" calculation is an estimate (to solve the economic problem); then the fluid dynamics system response must be accurately checked.

Once the variables are defined according to any possible practical restrictions for the selected "commercial" project, a detailed calculation (by small pipe sections or segments) shall be conducted, through appropriate programming of finite difference type (usually a steady state formulation considering heat transfer and then transient behavior checking) to verify the project parameters and define accurately the operation conditions and restrictions for the different components of the projected pipeline.

#### 4. Practical interpretation of obtained values

The obtained results follow a mathematical OPTIMISATION. The adoption of commercial or regulatory values gives way to a new cycle of OPTIMISATION possibilities in terms of commonly used or standardised variables (e.g. outer diameters of pipes), discrete variables (number of compressor stations), or dependant of regulations in force (maximum operating pressure), that is normally achieved through a series of detailed calculations of the gas pipeline system being designed.

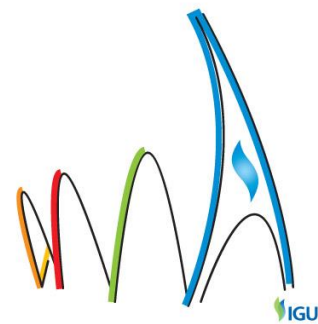
##### Maximum pressure $P_1$

If due to safety reasons or for technological limitations the optimum pressure  $P_1$  (or its equivalent  $e/D$  ratio) cannot be reached, the maximum possible pressure value will be adopted. If this maximum pressure differs greatly from the optimum value, a new OPTIMISATION round should be carried out, setting the possible pressure  $P_1$  and looking for the optimum values of diameter  $D$ , the compression ratio  $\left(\frac{P_1}{P_2}\right)$  and the number of compressor stations " $\chi$ ", as follows:

$$(V) \left( \frac{K_{02}K_{2t}}{\eta_m\eta_c} + \frac{K_3}{\eta_c} \right) C_\rho RT \left\{ \frac{1}{2} \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} \left[ \left( \frac{P_1}{P_2} \right)^2 - \frac{3K-1}{K-1} \right] + \frac{K}{K-1} \right\} - K_{03} = 0$$

$$(VI) \left[ K_{11} \frac{1+K_D}{10} \frac{\pi P_1 \gamma_f}{\sigma_{ad}} + \frac{4}{45} K_{04} K_4 K_D^2 \frac{\pi P_1}{RT} \right] D^7 + \left( \frac{K_{12} + K_{13}}{5} \pi \right) D^6 - \left( \frac{K_{02}K_{2t}}{\eta_m\eta_c} + \frac{K_3}{\eta_c} \right) C_\rho RT \left( \frac{P_1}{P_2} \right)^{\left(3-\frac{1}{K}\right)} \frac{8C_\rho^2 RTfL}{\pi^2 K_D^5 P_1^2} = 0$$

$$(VII) \chi = \frac{16C_\rho^2 RTf}{\pi^2 K_D^5 D^5 P_1^2 \left[ 1 - \left( \frac{P_1}{P_2} \right)^{-2} \right]} L$$



Then, practical alternatives are developed as above-explained, closing the complete optimisation calculation.

## 5. Conclusions

As shown when approaching the problem, the first step is to carry out a market research in order to obtain all the costs of the required components (materials, manpower, and equipment) that intervene in the facility, and then carry out a mathematical trend fitting to arrive at a cost.

Then, with the total cost function, the minimum cost within the border conditions set as per the flow equation (isothermal flow equation) is searched. This is done by using the Lagrange Multiplier Method, thereby finding the required solutions

Finally, optimum values have to be adjusted to practical parameters, that is diameter, maximum operation pressure and compression ratio for the compressor stations.

The proposed methodology strives to comply with the requirements of information, rationality and effective calculations possibilities, since most of the data used is already in the databases usually available in companies, even if some costs might eventually call for some limited market research.

Given the utmost importance that the gas pipeline networks have for the gas industry as a whole, all the effort put into achieving a greater economic rationale when designing a new system will never be too much.

The contents of this document are not the first approach to the subject<sup>4</sup>. The OPTIMISATION methods in the gas and oil industry in general, and in the fluid transportation field go back a long time. However, this new way of looking at this problem is an updated mathematical review, basically in the number of variables that are simultaneously optimised. The scope could be easily extended to different types of fluid transmission systems (crude oil lines, multi-product lines, water mains, slurry lines, etc.) and suited to local conditions regarding units, standards, usual values, etc.

## 6. References and Bibliography

The introduced methodology has been applied by the authors while carrying out the preliminary dimensioning of several fluid transportation lines in Argentina and Latin America, adapted for each case. Aires, by the authors, over 35 years of teaching activity at engineering schools. Moreover, this paper has been updated and extended on the basis of the paper submitted at the IGU-WGC TOKYO 2003.

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<sup>4</sup> See "Studio sui Gasodotti ad Alta Pressione" Pubblicazione de la Casa Clark Bros Co Traduzione ed edizione italiana curata dal Dott. Ing. Mariano Amico (Ed Faiilli-Roma circa 1950).